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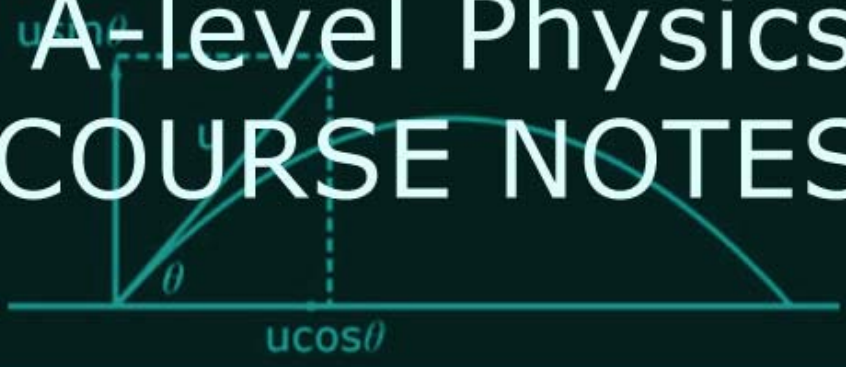
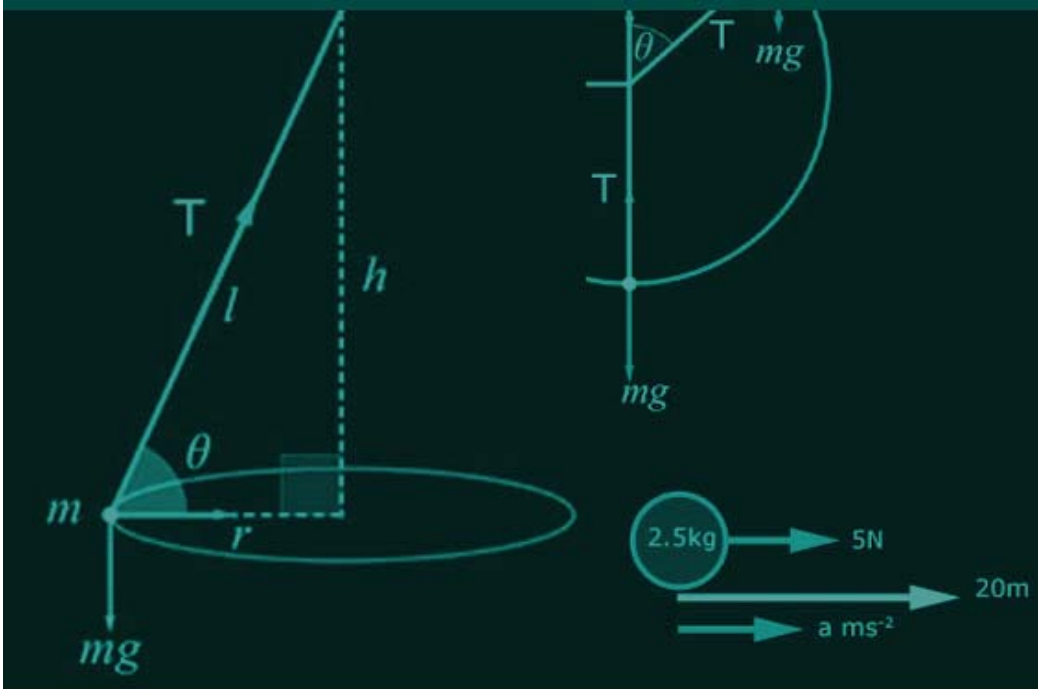


MECHANICS



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Linear Motion: Uniform Acceleration

Introduction

To understand this section you must remember the letters representing the variables:

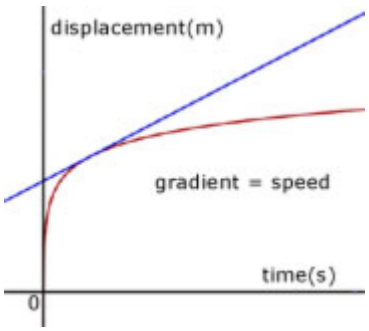
u - initial speed
v - final speed
a - acceleration(+) or deceleration(-)
t - time taken for the change
s - displacement(distance moved)

It is also important to know the **S.I. units** (*Le Système International d'Unités*) for these quantities:

u - metres per second (ms^{-1})
v - metres per second (ms^{-1})
a - metres per second per second (ms^{-2})
t - seconds (s)
s - metres (m)

in some text books 'speed' is replaced with 'velocity'. Velocity is more appropriate when direction is important.

Displacement-time graphs



For a displacement-time graph, the gradient at a point is equal to the **speed** .

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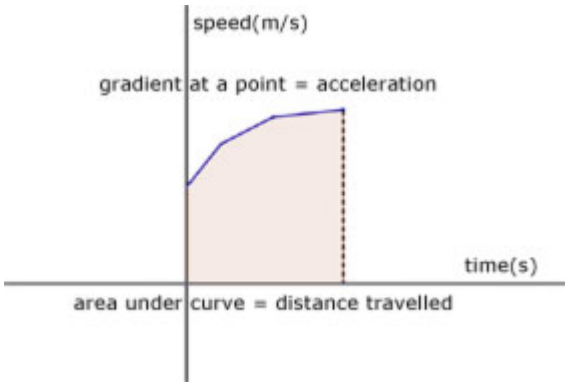
Speed-time graphs



For a speed-time graph, the area under the curve is the distance travelled.

The gradient at any point on the curve equals the acceleration.

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

Note, the acceleration is also the second derivative of a speed-time function.

Equations of Motion

One of the equations of motion stems from the definition of acceleration:

acceleration = the rate of change of speed

$$a = \frac{v-u}{t}$$

rearranging

$$v = u + at \qquad (1$$

if we define the distance 's' as the average speed times the time(t), then:

$$s = \left(\frac{u+v}{2}\right)t$$

rearranging

$$u + v = \frac{2s}{t}$$

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rearranging (i

$$v - u = at$$

subtracting these two equations to eliminate *v*

$$2u = \frac{2s}{t} - at$$
$$2ut = 2s - at^2$$
$$2ut + at^2 = 2s$$
$$ut + \frac{at^2}{2} = s, \qquad s = ut + \frac{at^2}{2}$$

it is left to the reader to show that :

$$v^2 - u^2 = 2as$$

hint: try multiplying the two equations instead of subtracting

summary:

$$s = \left(\frac{u+v}{2}\right)t$$
$$v - u = at$$
$$s = ut + \frac{at^2}{2}$$
$$v^2 - u^2 = 2as$$

Example #1

A car starts from rest and accelerates at 10 ms^{-1} for 3 secs.
What is the maximum speed it attains?

$$u = 0 \text{ ms}^{-1} \qquad a = 10 \text{ ms}^{-2} \qquad t = 3 \text{ s}$$

$$v = u + at$$
$$= 0 + (10 \times 3)$$
$$= 30$$

Ans. maximum speed is 30 ms^{-1}

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Example #2

A car travelling at 25 ms^{-1} starts to decelerate at 5 ms^{-2}.
How long will it take for the car to come to rest?

$$u = 25\ ms^{-1} \qquad v = 0 \qquad a = -5\ ms^{-2}$$

$$v = u + at$$
$$0 = 25 + (-5)t$$
$$5t = 25$$
$$t = 5$$

Ans. it takes 5 secs. for the car to stop

Example #3

A car travelling at 20 ms^{-1} decelerates at 5 ms^{-2}.
How far will the car travel before stopping?

$$u = 20\ ms^{-1} \qquad v = 0 \qquad a = -5\ ms^{-2}$$

$$v^2 - u^2 = 2as$$
$$0 - (20 \times 20) = 2(-5)s$$
$$-400 = -10s$$
$$s = \frac{400}{10} = 40$$

Ans. distance travelled is 40 m

Example #4

A car travelling at 30 ms^{-1} accelerates at 5 ms^{-2} for 8 secs.
How far did the car travel during the period of acceleration?

$$u = 30\ ms^{-1} \qquad a = 5\ ms^{-2} \qquad t = 8\ s$$

$$s = ut + \tfrac{1}{2}at^2$$

$$s = (30 \times 8) + \tfrac{1}{2}(5 \times 8 \times 8)$$
$$= 240 + 160$$
$$= 400$$

Ans. car travelled 400 m during acceleration

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Vertical motion under gravity

These problems concern a particle projected vertically upwards and falling 'under gravity'.

In these types of problem it is assumed that:

air resistance is minimal

displacement & velocity are positive(+) upwards & negative(-)downwards

acceleration(g) always acts downwards and is therefore negative(-)

acceleration due to gravity(g) is a constant

Example #1

A stone is thrown vertically upwards at 15 ms^{-1}.

(i) what is the maximum height attained?
(ii) how long is the stone in the air before hitting the ground?

(Assume g = 9.8 ms^{-2}. Both answers to 2 d.p.)

i)

$u = 15\ \text{ms}^{-1} \qquad a = -9.8\ \text{ms}^{-2}$

max. height is when final velocity $v = 0$

$$v^2 - u^2 = 2as$$
$$0 - (15)^2 = 2(-9.8)s$$
$$-225 = -19.6s$$
$$s = \frac{225}{19.6} = 11.4796$$

Ans. max. height is 11.48 m (2 d.p.)

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ii)

$u = 15\ \text{ms}^{-1} \qquad a = -9.8\ \text{ms}^{-2}$

time of flight is $2t$, twice time to max. height t
max. height is when final velocity $v = 0$

$$v = u + at$$
$$= 15 + (-9.8)t$$
$$15 = 9.8t$$
$$t = \frac{15}{9.8} = 1.5306$$
$$\therefore \quad 2t = 3.0612$$

Ans. max. time in air is 3.06 secs. (2 d.p.)

Example #2

A boy throws a stone vertically down a well at 12 ms^{-1}.
If he hears the stone hit the water 3 secs. later,

(i) how deep is the well?
(ii)what is the speed of the stone when it hits the water?

(Assume g = 9.8 ms^{-2}. Both answers to 1 d.p.)

i)

$u = -12\ \text{ms}^{-1} \qquad a = -9.8\ \text{ms}^{-2} \quad t = 3\ \text{s}$

$$s = ut + \tfrac{1}{2}at^2$$
$$= (-12)(3) + \tfrac{1}{2}(-9.8)(3\times 3)$$
$$= -36 - 44.1$$
$$= -80.1$$

Ans. depth of well is 80.1 m (1 d.p.)

ii)

$$v = u + at$$
$$= (-12) + (-9.8)(3)$$
$$= -121 - 29.4$$
$$= -41.4$$

Ans. stone strikes water at 41.4 ms^{-1} (1 d.p.)

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Linear Motion: Non-Uniform Acceleration

Theory

Consider a particle P moving in a straight line from a starting point O.

The displacement from O is x at time t .
The initial conditions are: $t \geq 0$ when $x=0$.
if v is the velocity of P at time t, then

$$v = \frac{dx}{dt}$$

The acceleration 'a' of particle P is defined as:

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

or alternately,

$$a = \frac{dv}{dt}$$
$$= \frac{dv}{dx}\frac{dx}{dt}$$

but $\frac{dx}{dt} = v$

$$\therefore \quad a = v\frac{dv}{dx}$$

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Problems on this topic are solved by analysing the information given to form a differential equation. This is then integrated, usually between limits.

Example #1

A particle moves in a straight line such that its acceleration 'a' at time 't' is given by:

$$a = 4t - 7$$

If the initial speed of the particle is 5 ms^{-1}, at what values of 't' is the particle stationary?

$$a = \frac{dv}{dt}$$

$$\Rightarrow \quad \frac{dv}{dt} = 4t - 7$$

$$\Rightarrow \quad dv = (4t - 7)dt$$

integrating both sides

$$\int dv = \int (4t - 7)dt$$

$$\Rightarrow \quad v = 2t^2 - 7t + C$$

but $v = 5\ ms^{-1}$ when $t = 0$

$$\Rightarrow \quad 5 = 0 - 0 + C$$

$$\therefore \quad C = 5$$

$$\Rightarrow \quad v = 2t^2 - 7t + 5$$

$$= (2t - 5)(t - 1)$$

the particle is at rest when $v = 0$

$$\therefore \quad (2t - 5)(t - 1) = 0$$

$$\Rightarrow \quad t = 5/2, \quad t = 1$$

particle is stationary at $t = 1$ sec. $t = 2.5$ sec.

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Example #2

A particle moves from a point O in a straight line with initial velocity 4 ms^{-1}.
if v is the velocity at any instant, the acceleration a of the particle is given by:

$$a = \frac{3}{v}$$

The particle passes through a point X with velocity 8 ms^{-1}.

(i) how long does the particle take to reach point X?
(ii) what is the distance OX?(1 d.p.)

i)

$$a = \frac{dv}{dt}$$

$$\Rightarrow \quad \frac{dv}{dt} = \frac{3}{v}$$

$$\Rightarrow \quad dt = \frac{v}{3}dv$$

integrating both sides

$$\int dt = \int \frac{v}{3}dv$$

the limits of v are 8 ms^{-1} and 4 ms^{-1}

$$t = \left[\frac{v^2}{6}\right]_4^8$$

$$= \left[\frac{64}{6}\right] - \left[\frac{16}{6}\right]$$

$$= \frac{48}{6} = 8$$

Ans. particle takes 8 secs. to reach X

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ii)

$$v\frac{dv}{dx} = \frac{3}{v}$$

$$\Rightarrow \qquad dx = \frac{v^2}{3}dv$$

integrating both sides

$$\int dx = \int \frac{v^2}{3}dv$$

the limits of v are 8ms^{-1} and 4ms^{-1}

$$x = \left[\frac{v^3}{9}\right]_4^8$$

$$= \left[\frac{512}{9}\right] - \left[\frac{64}{9}\right]$$

$$= \frac{448}{9} = 49.\dot{7}$$

Ans. distance OX is 49.8 metres (1 d.p.)

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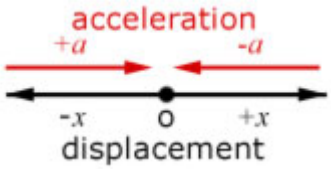
Linear Motion: Simple Harmonic Motion

Theory

A particle is said to move with S.H.M when the acceleration of the particle about a fixed point is proportional to its displacement but opposite in direction.



Hence, when the displacement is positive the acceleration is negative(and vice versa).

This can be described by the equation:

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

where x is the displacement about a fixed point O(positive to the right, negative to the left), and w^2 is a positive constant.

An equation for velocity is obtained using the expression for acceleration in terms of velocity and rate of change of velocity with respect to displacement(see 'non-uniform acceleration').

$$v\frac{dv}{dx} = -\omega^2 x$$

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separating the variable and integrating,

$$\int v\,dv = \int -\omega^2\,dx$$

$$\tfrac{1}{2}v^2 = -\tfrac{1}{2}\omega^2 x^2 + C$$

$v = 0$ when $x = a$

$$\Rightarrow \quad \underline{v^2 = \omega^2(a^2 - x^2)}$$

$$\Rightarrow \quad v = \pm\omega(a^2 - x^2)^{1/2}$$

but $$v = \frac{dx}{dt}$$

$$\Rightarrow \quad \frac{dx}{dt} = \pm\omega(a^2 - x^2)^{1/2}$$

separating the variable and integrating again,

$$\int \frac{dx}{(a^2 - x^2)^{1/2}} = \int \pm\omega\,dt$$

$$-\cos^{-1}\left(\frac{x}{a}\right) = \pm\omega t + C$$

when $t = 0$ when $x = a$

$$\Rightarrow \quad C = 0$$

$$\therefore \quad \cos(\omega t) = \frac{x}{a} \quad \text{or} \quad \underline{x = a\cos(\omega t)}$$

NB $\cos^{-1}()$ is the same as arc cos()

So the displacement against time is a cosine curve. This means that at the end of one completete cycle,

$$\omega T = 2\pi \qquad (T \text{ is the period})$$

$$\therefore \quad \underline{T = \frac{2\pi}{\omega}}$$

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Example

A particle displaying SHM moves in a straight line between extreme positions A & B and passes through a mid-position O.

If the distance AB=10 m and the max. speed of the particle is 15 m^{-1} find the period of the motion to 1 decimal place.

$AB = 10m \quad \therefore \quad \text{amplitude } a = 5m$

$\text{max. speed} \quad v_{max} = 15ms^{-1}$

$$v^2 = \omega^2(a^2 - x^2)$$

when v is max., displacement $x = 0$

$$\therefore \quad v^2 = \omega^2 a^2$$

$$\Rightarrow \quad v = \omega a$$

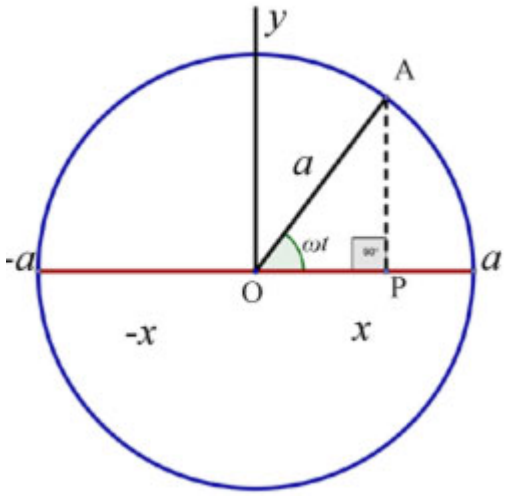
$$\Rightarrow \quad \omega = \frac{v}{a} = \frac{15}{5} = 3$$

period T is given by:

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{3} = 2.094$$

Ans. period of motion is 2.1 secs.

SHM and Circular Motion



The SHM-circle connection is used to solve problems concerning the time interval between particle positions.

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To prove how SHM is derived from circular motion we must first draw a circle of radius 'a'(max. displacement).

Then, the projection(x-coord.) of a particle A is made on the diameter along the x-axis. This projection, as the particle moves around the circle, is the SHM displacement about O.

from triangle OAP

$$x = a\cos\omega t \qquad (i$$

$$\frac{dx}{dt} = -a\omega\sin\omega t$$

$$\frac{d^2x}{dt^2} = -a\omega^2\cos\omega t$$

but $a\cos\omega t = x$ from (i

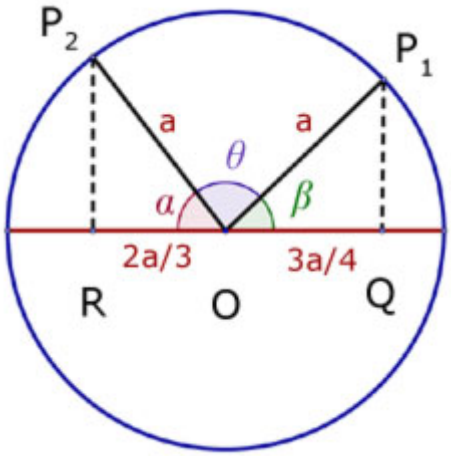
$$\Rightarrow \quad \frac{d^2x}{dt^2} = -\omega^2 x$$

Q.E.D. the motion of particle P along the x-axis is S.H.M.

Example

A particle P moving with SHM about a centre O, has period T and amplitude a .

Q is a point $3a/4$ from O. R is a point $2a/3$ from O.What is the time interval(in terms of T) for P to move directly from Q to R? Answer to 2 d.p.



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let the time interval between P_1 and P_2 be t secs.
let angle P_1OP_2 be θ rads.

period T is given by: $T = \frac{2\pi}{\omega}, \quad \omega = \frac{2\pi}{T}$ (i

$\Rightarrow \quad t = \frac{\theta}{\omega}$ (ii

the angles in a straight line = 180 deg.
$\therefore$ from the diagram, $\pi = \theta + \alpha + \beta$
$\Rightarrow \quad \theta = \pi - \alpha - \beta$

from (ii substituting for θ

$$t = \frac{\pi - \alpha - \beta}{\omega}$$

from (i substituting for ω

$$t = \frac{T(\pi - \alpha - \beta)}{2\pi} \quad \text{(iii}$$

from the diagram $\cos\alpha = \frac{\frac{2a}{3}}{a} = \frac{2}{3}$

$$\alpha = \cos^{-1}\left(\frac{2}{3}\right), \qquad \alpha = 0.8412$$

$$\cos\beta = \frac{\frac{3a}{4}}{a} = \frac{3}{4}$$

$$\beta = \cos^{-1}\left(\frac{3}{4}\right), \qquad \beta = 0.7227$$

substituting for α and β in (iii

$$t = \frac{T(\pi - 0.8412 - 0.7227)}{2\pi}$$

$$= \frac{T}{2\pi}(1.5777) = 0.2510T$$

Ans. time interval is $0.25T$

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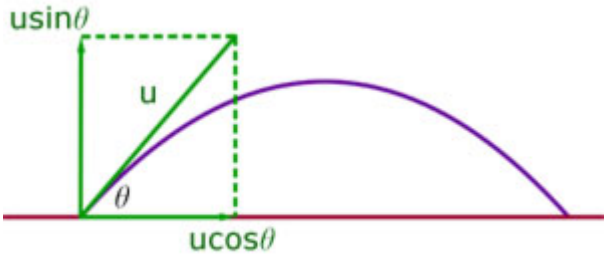
2D Motion: Projectiles

Vertical & horizontal components of velocity

When a particle is projected under gravity at a velocity u at an angle θ to the horizontal(neglecting air resistance)it follows the curve of a parabola.



The particle has an initial horizontal speed of ucosθ, which is unchanged throughout the motion.

Vertically the particle has an initial speed of usinθ. It falls under gravity and is accelerated downwards with an acceleration of g ms^{-1},where g = 9.8 ms^{-2} (approx.)

Time of flight

The time of flight is calculated from the vertical component of the velocity. It is the time it takes for the particle to go up, reach its maximum height and come down again. So this is twice the time to maximum height.

If the time to maximum height is t secs. , then the time of flight is 2t.

Consider motion up to maximum height. This is attained when the final velocity v = 0.

initial speed vertically upwards is $u\sin\theta$

using $v = u + at$

replacing u by $u\sin\theta$

substituting for acceleration $a = -g$

when $v = 0$

$$0 = u\sin\theta - gt$$

$$t = \frac{u\sin\theta}{g}$$

$\therefore$ time of flight$(2t)$ is $\dfrac{2u\sin\theta}{g}$

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Maximum height attained (H)

The maximum height attained occurs when the particle is momentarily stationary, before falling under gravity. The vertical component of speed is zero at this point(v=0).

using $v^2 - u^2 = 2as$
final speed $v = 0$
u is replaced with $u \sin\theta$
distance s is height H
substituting for acceleration $a = -g$

$$0 - u^2 \sin^2\theta = -2gH$$
$$-2gH = -u^2 \sin^2\theta$$

$$H = \frac{u^2 \sin^2\theta}{2g}$$

Range(R)

The range is simply the horizontal component of speed multiplied by the time of flight.

$$R = (u \cos\theta)t$$

Velocity(speed & direction) at time t

Solution of problems is to find the vertical component of speed at time t and combine this with the original horizontal component of speed, which remains unchanged.

Example
A particle P is projected at an angle of 45 degrees to the horizontal at a speed of 30 ms^{-1} . What is the speed and direction of the particle after 3 secs.?
(g=9.8 ms^{-2})

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constant horizontal speed $= 30\cos 45°$

$= 15.760 \text{ ms}^{-1}$

initial vertical speed $u = 30\cos 45°$

$= 15.760 \text{ ms}^{-1}$

v is vertical speed at $t = 3$ secs.

using $v = u + at$

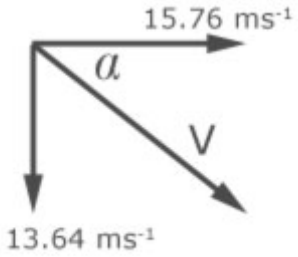
substituting for $a = -g$

$v = u - gt$

$= 15.76 - (9.8 \times 3)$

$= -13.64$

vertical component of speed is -13.64 ms^{-1}



using Pythgoras, the speed V at time t is given by,

$V^2 = (13.64)^2 + (15.76)^2$

$= 434.4272$

$\therefore \quad V = 20.8429$

speed of particle after 3 secs. is 20.84 ms^{-1}

if the speed is inclined α deg. to the horiz.

$\tan\alpha = \dfrac{13.64}{15.76} = 0.86548$

$\alpha = 40.8755°$

speed inclined at angle of 40.87° below horizontal

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2D Motion: Circular Motion

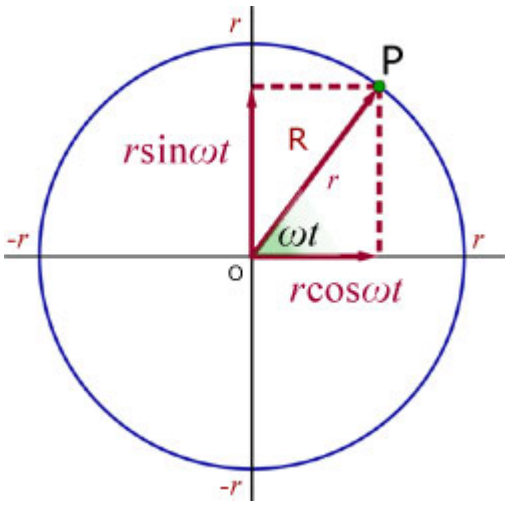
Summary of equations

$$\omega = \frac{\theta}{t} \qquad \omega = \frac{v}{r}$$

$$|\text{radial acceleration}| = \omega^2 r = \frac{v^2}{r}$$

$$|\text{tangential acceleration}| = \frac{dv}{dt}$$

Describing the circle - position vector **R**



i & **j** are unit vectors along the x and y-axis respectively.

The position vector **R** of a particle at P from O, at time t, is given by:

$$\mathbf{R} = (r\cos\omega t)\mathbf{i} + (r\sin\omega t)\mathbf{j}$$

As the position vector **R** rotates anti-clockwise, the particle at P traces out a circle of radius r .

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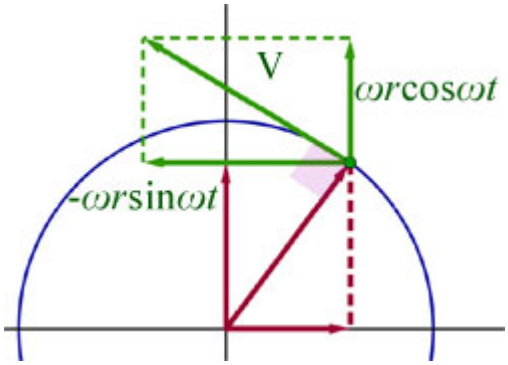
The velocity vector **V**

The velocity vector **V** at an instant is given by differentiating the position vector **R** with respect to t .

Here the unit vectors **i** & **j**, parallel to the x and y-axes, are centred on the particle at P.

$$\frac{d\mathbf{R}}{dt} = \mathbf{V}$$

$$\mathbf{V} = (-\omega r \sin \omega t)\mathbf{i} + (\omega r \cos \omega t)\mathbf{j}$$



the magnitude of the velocity is given by:

$$|\mathbf{V}| = \sqrt{(-\omega r \sin \omega t)^2 + (\omega r \cos \omega t)^2}$$
$$= \sqrt{\omega^2 r^2 \sin^2 \omega t + \omega^2 r^2 \cos^2 \omega t}$$
$$= \sqrt{\omega^2 r^2 (\sin^2 \omega t + \cos^2 \omega t)}$$

but $\sin^2 \omega t + \cos^2 \omega t = 1$

$$\therefore \quad |\mathbf{V}| = \sqrt{\omega^2 r^2}$$

$$\underline{|\mathbf{V}| = \omega r}$$

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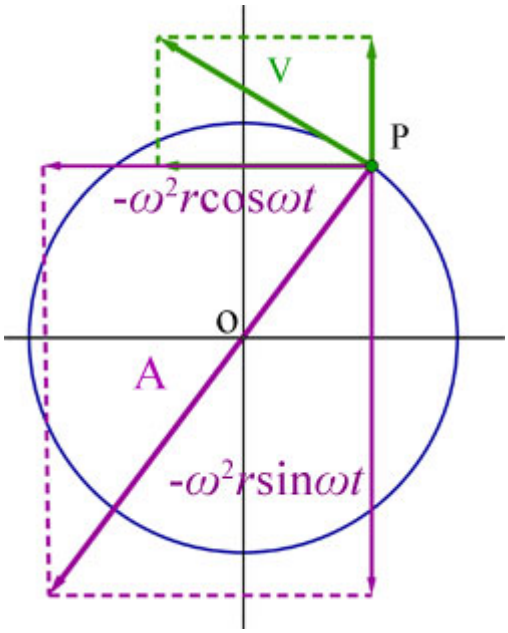
The acceleration vector **A**

The acceleration of the particle at P is given by differentiating **V** with respect to t .



$$\mathbf{A} = \frac{d\mathbf{V}}{dt}$$

$$= (-\omega^2 r\cos\omega t)\mathbf{i} + (-\omega^2 r\sin\omega t)\mathbf{j}$$

but $\mathbf{R} = (r\cos\omega t)\mathbf{i} + (r\sin\omega t)\mathbf{j}$

$$\therefore \quad \mathbf{A} = -\omega^2\mathbf{R}$$

$$\Rightarrow \quad \underline{|\mathbf{A}| = -\omega^2 r}$$

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Example

A satellite is moving at 2000 ms^{-1} in a circular orbit around a distant moon.
If the radius of the circle followed by the satellite is 1000 km, find:
i) the acceleration of the satellite
ii) the time for the satellite to complete one full orbit of the moon in minutes(2d.p.).

i) a is acceleration, v speed and r orbit radius,

$$a = \frac{v^2}{r}$$

substituting for $v = 2000\ ms^{-1}$, $r = 1000\ km(10^6 m)$

$$a = \frac{2000 \times 2000}{1000000} = 4$$

Ans. acceleration of the satellite is 4 ms^{-2}

ii) distance travelled by satellite in one orbit

= circumference of orbit circle

$$= 2\pi r = 2\pi \times 10^6$$

$$\text{time for one orbit} = \frac{\text{dist. travelled in one orbit}}{\text{speed}, v}$$

$$= \frac{2\pi \times 10^6}{2 \times 10^3} = 10^3 \pi$$

$$= 3142 \text{ secs.} \equiv 52.3\dot{6} \text{ mins.}$$

Ans. time for one orbit is $52.3\dot{6}$ mins.

Non-uniform circular motion(vertical circle)

A more in-depth treatment of motion in a vertical circle is to be found in 'kinetics/more circular motion'.

Here we look at the more general case of the acceleration component along the circle and the component towards the centre varying.

$$a_{\text{towards centre}} = \frac{v^2}{r}$$

$$a_{\text{along tangent}} = \frac{dv}{dt}$$

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Example: A particle starts moving in a circular direction with angular speed of 5 rad s^{-1} . The radius of the circle of motion is 4 m, and the angular speed at time *t* is given by,

$$\omega = 15 - 3t$$

What is, i) the linear speed of the particle 6 secs. after it starts moving?

ii) the resultant particle acceleration? (answers to 1 d.p.)

i) $r = 4$ m $\quad t = 6$ secs.

$\omega = \dfrac{v}{r}, \quad v = \omega r$

substituting for $\omega = 15 - 3t$

$v = r(15 - 3t)$

$= 4(15 - 3 \times 6) = 4(15 - 18) = 4 \times (-3)$

$= -12$

Ans. linear speed of particle is 12.0 ms^{-1} (1d.p.)

ii) tangential acceleration component $= \dfrac{dv}{dt}$

$\dfrac{dv}{dt} = \dfrac{d(\omega r)}{dt} = r\dfrac{d\omega}{dt}$

substituting for $\omega = 15 - 3t$

$\dfrac{dv}{dt} = r\dfrac{d(15 - 3t)}{dt} = -3r$

substituting for $r = 4$ m

$\dfrac{dv}{dt} = -3 \times 4 = -12$ ms^{-2}

radial acceleration component $= \dfrac{v^2}{r}$

$\dfrac{v^2}{r} = \dfrac{12 \times 12}{4} = \dfrac{144}{4} = 36$ ms^{-2}

resultant acceleration R (using Pythagoras),

$R = \sqrt{(-12)^2 + (36)^2} = \sqrt{144 + 1296} = 37.9$

Ans. resultant acceleration is magnitude 37.9 ms^{-2}

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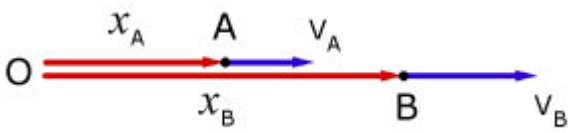
2D Motion: Relative Motion

One dimensional relative velocity(in a line)

Consider two particles A and B at instant t positioned along the x-axis from point O.

Particle A has a displacement $\boldsymbol{x}_A$ from O, and a velocity $\mathbf{V}_A$along the x-axis. The displacement $\boldsymbol{x}_A$ is a function of time t .

Particle B has a displacement $\boldsymbol{x}_B$ from O, and a velocity $\mathbf{V}_B$along the x-axis. The displacement $\boldsymbol{x}_B$ is a function of time t .



The velocity $\mathbf{V}_B$ relative to velocity $\mathbf{V}_A$ is written,

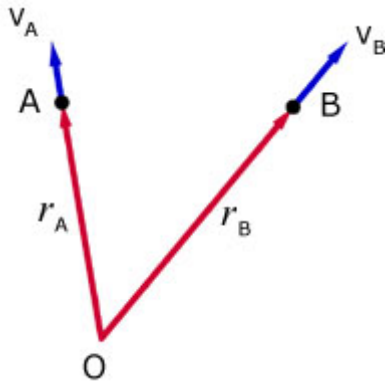
$$_B\mathbf{V}_A = \mathbf{V}_B - \mathbf{V}_A$$

This can be expressed in terms of the derivative of the displacement with respect to time.

$$\mathrm{V}_B = \frac{d(\boldsymbol{x}_B)}{dt} \qquad \mathrm{V}_A = \frac{d(\boldsymbol{x}_A)}{dt}$$

$$_B\mathrm{V}_A = \frac{d(\boldsymbol{x}_B)}{dt} - \frac{d(\boldsymbol{x}_A)}{dt}$$

Two dimensional relative position & velocity



Particle A has a displacement $\boldsymbol{r}_A$ from O, and a velocity $\mathbf{V}_A$along the x-axis. The displacement $\boldsymbol{r}_A$ is a function of time t .

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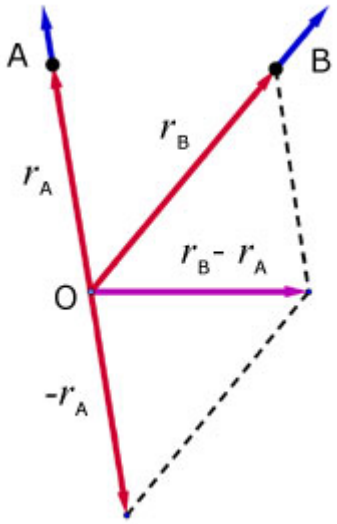
Particle A has a displacement $\boldsymbol{r}_B$ from O, and a velocity $\mathbf{V}_B$along the x-axis. The displacement $\boldsymbol{r}_B$ is a function of time t .

Relative position

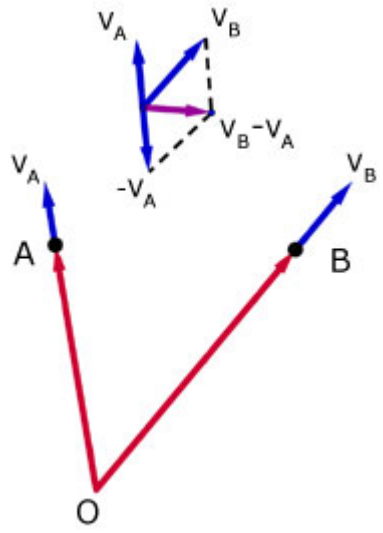


The position of B relative to A at time t is given by the position vector from O, $\boldsymbol{r}_{B-A}$.

The position vector $\boldsymbol{r}_{B-A}$ can be written as,

$$_B\boldsymbol{r}_A = \boldsymbol{r}_B - \boldsymbol{r}_A$$

Relative velocity



Similarly, at time t the velocity vector $\mathbf{V}_B$ relative to velocity vector $\mathbf{V}_A$ can be written,

$$_B\mathbf{V}_A = \mathbf{V}_B - \mathbf{V}_A$$

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This can be expressed in terms of the derivative of the displacement with respect to time.

$$\mathbf{V}_B = \frac{d(r_B)}{dt} \qquad \mathbf{V}_A = \frac{d(r_A)}{dt}$$

$${}_B\mathbf{V}_A = \frac{d(r_B)}{dt} - \frac{d(r_A)}{dt}$$

Example #1

If the velocity of a particle P is (9**i** - 2**j**) ms^{-1} and the velocity of another particle Q is (3**i** - 8**j**) ms^{-1} , what is the velocity of particle P relative to Q?

$$\begin{aligned} {}_P\mathbf{V}_Q &= \mathbf{V}_P - \mathbf{V}_Q \\ &= (9\boldsymbol{i} - 2\boldsymbol{j}) - (3\boldsymbol{i} - 8\boldsymbol{j}) \\ &= 9\boldsymbol{i} - 3\boldsymbol{i} + 8\boldsymbol{j} - 2\boldsymbol{j} \\ &= (9\boldsymbol{i} - 3\boldsymbol{i}) + (8\boldsymbol{j} - 2\boldsymbol{j}) \\ &= 6\boldsymbol{i} + 6\boldsymbol{j} \end{aligned}$$

Ans. velocity of P relative to Q is $(6\boldsymbol{i} + 6\boldsymbol{j})\ ms^{-1}$

Example #2

A particle P has a velocity (4**i** + 3**j**) ms^{-1}. If a second particle Q has a relative velocity to P of (2**i** - 3**j**), what is the velocity of Q?

$$\begin{aligned} {}_Q\mathbf{V}_P &= \mathbf{V}_Q - \mathbf{V}_P \\ \mathbf{V}_Q &= {}_Q\mathbf{V}_P + \mathbf{V}_P \\ &= (2\boldsymbol{i} - 3\boldsymbol{j}) + (4\boldsymbol{i} + 3\boldsymbol{j}) \\ &= 2\boldsymbol{i} + 4\boldsymbol{i} - 3\boldsymbol{j} + 3\boldsymbol{j} \\ &= 6\boldsymbol{i} \end{aligned}$$

Ans. velocity of Q is $(6\boldsymbol{i})\ ms^{-1}$

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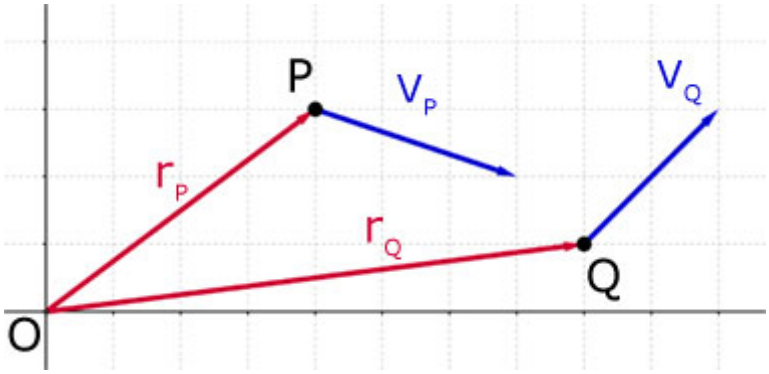
Example #3

A radar station at O tracks two ships P & Q at 0900hours (t=0) .
P has position vector (4**i** + 3**j**) km, with velocity vector (3**i** - **j**) km hr $^{-1}$.
Q has position vector (8**i** + **j**) km, with velocity vector (2**i** + 2**j**) km hr $^{-1}$.

i) What is the displacement of P relative to Q at 0900 hours? (ie distance between ships). Answer to 2 d.p.
ii) Write an expression for the displacement of P relative to Q in terms of time t .
iii) Hence calculate the displacement of P relative to Q at 1500 hours.
iv) At what time are the two ships closest approach and what is the distance between them at this time?



i)

$$\boldsymbol{r}_P = (4\boldsymbol{i} + 3\boldsymbol{j}) \qquad \boldsymbol{r}_Q = (8\boldsymbol{i} + \boldsymbol{j})$$

$$\begin{aligned}\boldsymbol{r}_P - \boldsymbol{r}_Q &= (4\boldsymbol{i} + 3\boldsymbol{j}) - (8\boldsymbol{i} + \boldsymbol{j}) \\ &= 4\boldsymbol{i} + 3\boldsymbol{j} - 8\boldsymbol{i} - \boldsymbol{j} \\ &= 4\boldsymbol{i} - 8\boldsymbol{i} - \boldsymbol{j} + 3\boldsymbol{j} \\ &= (-4\boldsymbol{i} + 2\boldsymbol{j})\end{aligned}$$

magnitude of the displacement, using Pythagoras,

$$\begin{aligned}\left|\boldsymbol{r}_P - \boldsymbol{r}_Q\right| &= \sqrt{(-4)^2 + (2)^2} \\ &= \sqrt{16 + 4} = \sqrt{20} = 4.47\end{aligned}$$

Ans. distance between ships at 0900 is 4.47 km.

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ii)

$\boldsymbol{r}_{P,0}$ displacement vector of P at time 0

$\boldsymbol{r}_{P,t}$ displacement vector of P at time t

$\boldsymbol{r}_{Q,0}$ displacement vector of Q at time 0

$\boldsymbol{r}_{Q,t}$ displacement vector of Q at time t

$|V_P t|$ distance travelled by P in time t

$|V_Q t|$ distance travelled by Q in time t

$$\begin{aligned}\boldsymbol{r}_{P,t} &= \boldsymbol{r}_{P,0} + V_P t \\ &= (4\boldsymbol{i} + 3\boldsymbol{j}) + (3\boldsymbol{i} - \boldsymbol{j})\end{aligned}$$

$$\begin{aligned}\boldsymbol{r}_{Q,t} &= \boldsymbol{r}_{Q,0} + V_Q t \\ &= (8\boldsymbol{i} + \boldsymbol{j}) + (2\boldsymbol{i} - 2\boldsymbol{j})\end{aligned}$$

therefore the displacement of P relative to Q is given by,

$$\begin{aligned}{}_{P,t}\boldsymbol{r}_{Q,t} &= \boldsymbol{r}_{P,t} - \boldsymbol{r}_{Q,t} \\ &= (4\boldsymbol{i} + 3\boldsymbol{j}) + (3\boldsymbol{i} - \boldsymbol{j})t - [(8\boldsymbol{i} + \boldsymbol{j}) + (2\boldsymbol{i} + 2\boldsymbol{j})t] \\ &= 4\boldsymbol{i} + 3\boldsymbol{j} + 3\boldsymbol{i}t - \boldsymbol{j}t - 8\boldsymbol{i} - \boldsymbol{j} - 2\boldsymbol{i}t - 2\boldsymbol{j}t \\ &= (4\boldsymbol{i} - 8\boldsymbol{i}) + (3\boldsymbol{j} - \boldsymbol{j}) + 3\boldsymbol{i}t - \boldsymbol{j}t - 2\boldsymbol{i}t - 2\boldsymbol{j}t \\ &= (-4\boldsymbol{i} + 2\boldsymbol{j}) + (3\boldsymbol{i} - 2\boldsymbol{i} - \boldsymbol{j} - 2\boldsymbol{j})t \\ &= (-4\boldsymbol{i} + 2\boldsymbol{j}) + (\boldsymbol{i} - 3\boldsymbol{j})t\end{aligned}$$

$$\underline{{}_{P,t}\boldsymbol{r}_{Q,t} = (-4\boldsymbol{i} + 2\boldsymbol{j}) + (\boldsymbol{i} - 3\boldsymbol{j})t}$$

iii) using the result above for 1500 hours(t = 6)

$$\begin{aligned}{}_{P,t}\boldsymbol{r}_{Q,t} &= (-4\boldsymbol{i} + 2\boldsymbol{j}) + (\boldsymbol{i} - 3\boldsymbol{j})6 \\ &= -4\boldsymbol{i} + 2\boldsymbol{j} + 6\boldsymbol{i} - 18\boldsymbol{j} \\ &= -4\boldsymbol{i} + 6\boldsymbol{i} + 2\boldsymbol{j} - 18\boldsymbol{j} \\ &= 2\boldsymbol{i} - 16\boldsymbol{j}\end{aligned}$$

$$\begin{aligned}\left|{}_{P,t}\boldsymbol{r}_{Q,t}\right| &= \sqrt{(2)^2 + (16)^2} \\ &= \sqrt{4 + 256} = \sqrt{260} = 16.12\end{aligned}$$

Ans. displacement P relative Q at 1500 hours is 16.12 km

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iv) Closest approach is when the position vector of P is at right angles to the reference vector.

The 'reference vector' is the first part of the vector equation for $\boldsymbol{r}$.

The position vector gives the point P at time t along the straight line described by the vector equation.

Two dimensional relative acceleration

Similarly, if $\mathbf{a}_A$ and $\mathbf{a}_B$ are the acceleration vectors at A and B at time t, then the acceleration of B relative to A is given by,

$$\mathbf{a}_B = \frac{d(\mathbf{V}_B)}{dt} \qquad \mathbf{a}_A = \frac{d(\mathbf{V}_A)}{dt}$$

$$_B\mathbf{a}_A = \frac{d(\mathbf{V}_B)}{dt} - \frac{d(\mathbf{V}_A)}{dt}$$

$$= \frac{d^2(\mathbf{r}_B)}{dt^2} - \frac{d^2(\mathbf{r}_A)}{dt^2}$$

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Kinetics: Newton's Laws of Motion

Newton's Laws of Motion

1. A body will remain at rest or travel at uniform linear velocity unless acted upon by an external force.

2. The rate of change of linear momentum is proportional to the applied force and acts in the same direction as the force.

3. The forces of two bodies on each other are equal and directed in opposite directions.

Definition of momentum

Momentum(**P**) is a vector quantity equal in magnitude to the product of mass and velocity.

Note, mass(m) is a scalar quantity, while velocity(**v**) is a vector quantity.

$$\mathbf{P} = m\mathbf{v}$$

m (kg) $\quad$ **v**(ms^{-1}) $\quad$ **P** (kg. ms^{-1})

The letter 'p' in small case is designated to represent pressure.

Theory

If we consider a force **F** acting on a mass m with velocity **v**, the Second law may be represented by the proportionality:

$$\mathbf{F} \propto \frac{d(m\mathbf{v})}{dt}$$

$$\mathbf{F} = k\frac{d(m\mathbf{v})}{dt}$$

$$= k\,m\frac{d(\mathbf{v})}{dt}$$

but acceleration $\quad \mathbf{a} = \frac{d(\mathbf{v})}{dt}$

$$\Rightarrow \quad \mathbf{F} = km\mathbf{a}$$

making $\quad \mathbf{F} = 1$ Newton, $\quad m = 1$ kg. $\quad \mathbf{a} = 1ms^{-2}$

$$\Rightarrow \quad k = 0$$

$$\Rightarrow \quad \mathbf{F} = m\mathbf{a} \qquad \text{force = mass x acceleration}$$

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The Newton N

As seen from the theory relating to the Second Law, to get rid of the constant of proportionality each quantity is made unity.

So we come to our definition of a Newton:

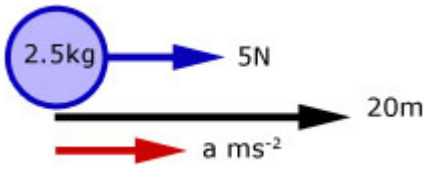
A **Newton** is the force that when applied to a **1 kg mass** will give it an **acceleration of 1 ms^{-2}**.

Linear acceleration

Here the mass is either stationary and is accelerated by a force in a straight line or is initially moving at constant velocity before the force is applied.

Example #1

A 5N force acts on a 2.5kg mass, making it accelerate in a straight line.
i) What is the acceleration of the mass?
ii) How long will it take to move the mass through 20m?
(Answer to 2 d.p.)



i)

$F = 5\text{N} \quad m = 2.5\text{kg} \quad s = 20\text{m} \quad u = 0\text{ms}^{-1}$

using $F = ma$

$$\Rightarrow \quad a = \frac{F}{m}$$

$$= \frac{5}{2.5} = 2$$

Ans. acceleration is $2\ \text{ms}^{-1}$

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ii)

$$\text{using} \qquad s = ut + \tfrac{1}{2}at^2$$

$$u = 0\text{ms}^{-1} \quad \Rightarrow \quad s = \tfrac{1}{2}at^2$$

$$t^2 = \frac{2s}{a}$$

$$t = \sqrt{\frac{2s}{a}}$$

$$s = 20\text{m} \qquad a = 2\text{ms}^{-2}$$

$$t = \sqrt{\frac{2 \times 20}{2}} = \sqrt{20} = 4.47$$

Ans. time for mass to move 20 m is 4.47 secs.

Example #2

A force causes a 3kg mass to accelerate. If the velocity of the mass at time t is given by:

$$\mathbf{v} = 2t\mathbf{i} - 3t^2\mathbf{j} + 4t^3\mathbf{k}$$

what is the magnitude of the force when t = 5 secs. ?

$$\text{acceleration} \quad \mathbf{a} = \frac{d\mathbf{v}}{dt}$$

$$\frac{d\mathbf{v}}{dt} = 2\mathbf{i} - 6t\mathbf{j} + 12t^2\mathbf{k}$$

$$\text{using} \quad \mathbf{F} = m\mathbf{a}$$

$$= m(2\mathbf{i} - 6t\mathbf{j} + 12t^2\mathbf{k})$$

$$\text{substituting for} \quad m = 3\text{ kg}, \quad t = 5\text{ secs.}$$

$$\mathbf{F} = 3(2\mathbf{i} - 30\mathbf{j} + 300\mathbf{k})$$

$$= 6\mathbf{i} - 90\mathbf{j} + 900\mathbf{k})$$

hence the magnitude of $\mathbf{F}$ is given by:

$$|\mathbf{F}| = \sqrt{(6)^2 + (-90)^2 + (900)^2}$$

$$= \sqrt{36 + 8100 + 810000}$$

$$= \sqrt{818136} = 904.51$$

Ans. when t = 5 secs. applied force is 904.51 N

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Linear retardation

Here the mass is already moving at constant velocity in a straight line before the force is applied, opposing the motion.

Example #1

A 4 kg mass travelling at constant velocity 15 ms^{-1} has a 10 N force applied to it against the direction of motion.
i) What is the deceleration produced?
ii) How long will it take before the mass is brought to rest?

i)

$$u = 15\ \text{ms}^{-1} \quad m = 4\ \text{kg} \quad F = 10\ \text{N}$$

$$F = ma$$

$$a = \frac{F}{m}$$

$$a = \frac{10}{4} = 2.5$$

Ans. deceleration is 2.5 ms^{-1}

ii)

$$v = 0$$

$$v = u - at$$

$$0 = u - at$$

$$at = u$$

$$t = \frac{u}{a} = \frac{15}{2.5} = 6$$

Ans. mass brought to rest in 6 secs.

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Example #2

A sky diver with mass 80kg is falling at a constant velocity of 70 ms^{-1} . When he opens his parachute he experiences a constant deceleration of 3g for 2 seconds.
i) What is the magnitude of the decelerating force?
ii) What is his rate of descent at the end of the 2 seconds deceleration?

i)

$$m = 80 \text{ kg} \qquad u = 70 \text{ ms}^{-1} \quad a = 3g$$

$$\begin{aligned} F &= ma \\ &= 80 \times 3 \times 9.8 \\ &= 2352 \end{aligned}$$

Ans. decelerating force is 2352 N

ii)

$$u = 70 \text{ ms}^{-1} \quad a = 3 \times 9.8 = 29.4 \text{ ms}^{-2} \quad t = 2 \text{ secs.}$$

$$\begin{aligned} v &= u - at \\ &= 70 - (29.4) \times (2) \\ &= 11.2 \end{aligned}$$

Ans. final rate of descent is 11.2 ms^{-1}

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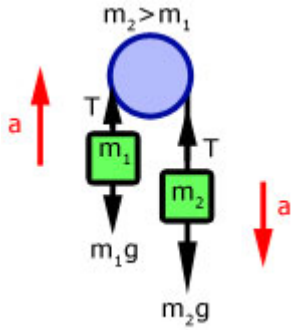
Kinetics: Connected Particles

Pulleys

Problems involve two weights either side of a pulley. The heavier weight pulls on the lighter causing both to accelerate in one direction with a common acceleration.



calculation of acceleration 'a'

$m_2 > m_1$

$$T - m_1 g = m_1 a$$

$$m_2 g - T = m_2 a$$

$$m_2 g - m_1 g = (m_1 + m_2) a$$

$$a = \frac{(m_2 - m_2)}{(m_1 + m_2)} g$$

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calculation of tension 'T'

$m_2 > m_1$

$$T - m_1 g = m_1 a$$

$$a = \frac{T - m_1 g}{m_1} \qquad \text{(i}$$

$$m_2 g - T = m_2 a$$

$$a = \frac{m_2 g - T}{m_2} \qquad \text{(ii}$$

equating (i and (ii

$$\frac{T - m_1 g}{m_1} = \frac{m_2 g - T}{m_2}$$

$$m_2 T - m_1 m_2 g = m_1 m_2 g - m_1 T$$

$$m_2 T + m_1 T = m_1 m_2 g + m_1 m_2 g$$

$$T(m_2 + m_1) = 2 m_1 m_2 g$$

$$\underline{T = \frac{2 m_1 m_2 g}{(m_2 + m_1)}}$$

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Example

A 3 kg mass and a 5 kg mass are connected over a pulley by a light inextensible string. When the masses are released from rest, what is:
i) the acceleration of each mass?
ii) the tension in the string
(Take g=9.8 ms^{-2} . Answer to 2 d.p.)

i)

$m_1 = 3\text{ kg} \quad m_2 = 5\text{ kg} \quad g = 9.8\text{ ms}^{-2}$

for mass m_1, using $f = ma$

$$T - m_1 g = m_1 a$$

$$T - 3g = 3a \qquad \text{(i}$$

for mass m_2, using $f = ma$

$$m_2 g - T = m_2 a$$

$$5g - T = 5a \qquad \text{(ii}$$

adding equations (i and (ii

$$T - 3g = 3a$$

$$\underline{5g - T = 5a}$$

$$2g = 8a$$

$$a = \frac{g}{4} = \frac{9.8}{4} = 2.45$$

Ans. acceleration of each mass is 2.45 ms^{-2}

ii)

from equation (i above

$$T - 3g = 3a$$

$$T = 3a + 3g$$

substituting for the values of g and a

$$T = (3 \times 2.45) + (3 \times 9.8) = 36.75$$

Ans. the tension in the string is 36.75 N

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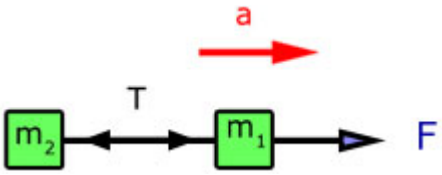
Tow-bar/tow-rope/chains

Usually one body pulled horizontally by another with each linked by a tow-bar or similar. This is similar to the pulley but drawn out in a line.



assuming no friction,

calculation of acceleration 'a'

$$T = m_2 a \qquad \text{(i}$$

$$F - T = m_1 a \qquad \text{(ii}$$

adding (i and (ii

$$F = (m_1 + m_2)a$$

$$a = \frac{F}{(m_1 + m_2)}$$

calculation of tension 'T'

from (i $T = m_2 a$

substituting for a

$$T = \frac{m_2 F}{(m_1 + m_2)}$$

Example

A car of mass 600 kg towes a trailer of mass 250 kg in a straight line using a rigid towe-bar.
The resistive force on the car is 200N.
The resistive force on the trailer is 80N.
If the forward thrust produced by the engine of the car is 800 N, what is(to 3 d.p.)
i) the acceleration of the car
ii) the tension in the tow-bar

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i) looking at all the forces on the car and trailer together

thrust - resistance forces = total mass × acceleration
(tensions forward and backwards cancel out)

$$800 - 80 - 200 = (600 + 250)a$$
$$520 = 850a$$
$$a = \frac{520}{850} = 0.612$$

Ans. acceleration of car is 0.612 ms^{-2}

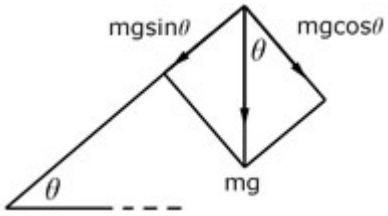
ii)

using $f = ma$ for the trailer only
$$T - 80 = 250a$$
$$T = 250a + 80$$
$$T = (250 \times 0.612) + 80 = 233$$

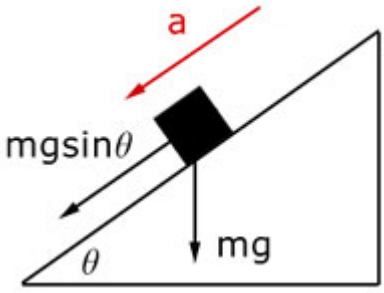
Ans. tension in towe-bar is 233N

Inclined plane with pulley

The pulley at the end just changes the direction of the force. problems involve the resolved component of the weight of the object down the plane.



for a mass sliding down a smooth incline



$$mg\sin\theta = ma$$

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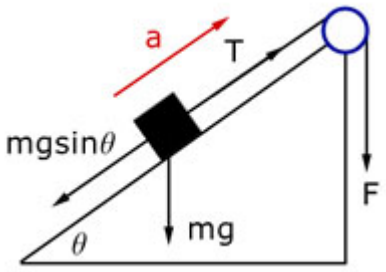
for a mass pulled up an incline via a pulley



$$F - mg\sin\theta = ma$$

Example

A 2 kg mass on a smooth 30° plane is connected to a 5 kg mass by a light inextensible string passing over a pulley at the top of the plane.
When the particles are released from rest the 2 kg mass moves up the plane.
i) what is the acceleration of the 2 kg & 5 kg masses?
ii) What is the tension in the string?

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i)

T tension in string

m_1 mass being pulled up incline

m_2 mass falling under gravity

a acceleration of both masses

$m_1 = 2$ kg $\quad m_2 = 5$ kg $\quad g = 9.8\ \text{ms}^{-2}$

consider forces on the 2kg mass, using $f = ma$

$$T - m_1 g \sin 30^\circ = m_1 a$$

$$T - (2 \times 9.8 \times 0.5) = 2a$$

$$T - 9.8 = 2a \qquad \text{(i}$$

consider forces on the kg mass, using $f = ma$

$$m_2 g - T = m_2 a$$

$$(5 \times 9.8) - T = 5a$$

$$49 - T = 5a \qquad \text{(ii}$$

adding equations (i and (ii

$$T - 9.8 = 2a$$

$$\underline{49 - T = 5a}$$

$$49 - 9.8 = 7a$$

$$a = \frac{39.2}{7} = 5.6$$

Ans. acceleration of the 2 kg & 5kg masses is 5.6 ms^{-2}

ii)

substituting into equation (i for a

$$T - 9.8 = 2a$$

$$T = 2a + 9.8$$

$$= (2 \times 5.6) + 9.8 = 21$$

Ans. tension in string is 21N

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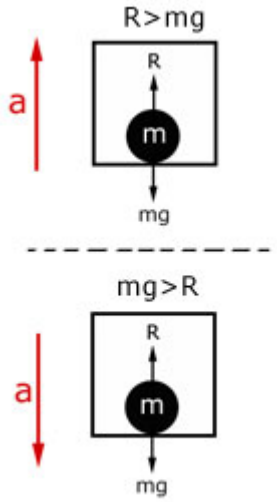
Mass ascending or descending in a lift

it is important to remember that there are only two forces on the body in the lift - the weight down and the reaction of the floor up.



Example

A person of mass 100 kg stands in a lift.
What is the force exerted by the lift floor on the person when the lift is:
i) moving upwards at 3 ms^{-1}
ii) moving downwards at 4 ms^{-1}

i)

R reaction upwards of the lift floor
a acceleration of the lift upwards
m mass of person
$m = 100$ kg $\quad a = 3 ms^{-1}$ $\quad g = 9.8\ ms^{-1}$
considering movement upwards, using $f = ma$

$$R - mg = ma$$
$$R = ma + mg$$
$$= (100 \times 3) + (100 \times 9.8)$$
$$= 300 + 980 = 1280$$

Ans. reaction of floor, lift ascending, is 1280 N

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ii)

a acceleration of the lift downwards

$a = 4\ \text{ms}^{-1}$

considering movement downwards, using $f = ma$

$$mg - R = ma$$

$$R = mg - ma$$

$$= (100 \times 9.8) - (100 \times 4)$$

$$= 980 - 400 = 580$$

Ans. reaction of floor, lift descending, is 580 N

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Kinetics: Work & Energy

Theory - Work & Energy

Consider a particle of mass m moving linearly with an applied force F constantly acting on it.
u = initial speed, v = final speed, a = acceleration,
s = distance covered, t = time taken

from Newton's 2nd Law,

$$F = ma \qquad \text{(i}$$

from equations describing uniform linear motion

$$v^2 - u^2 = 2as$$

$$\Rightarrow \qquad a = \frac{v^2 - u^2}{2s}$$

substituting for a in equation (i

$$F = m\left(\frac{v^2 - u^2}{2s}\right)$$

$$= \frac{1}{s}\left[\tfrac{1}{2}mv^2 - \tfrac{1}{2}mu^2\right]$$

$$\underline{Fs = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mu^2}$$

By definition,

work done = (force) x (distance force moves)

Since the expression $\frac{1}{2}mv^2$ is defined as the kinetic energy of a particle of mass m, speed v, our definition is modified to:

work done = change in kinetic energy produced

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mathematical proof

Consider a particle, mass m, speed v, being moved along the x-axis by a force of magnitude F.
The applied force F is proportional to the displacement of the particle, x, along the x-axis.

$$F = ma$$

since $$a = v\frac{dv}{dx}$$

$$\Rightarrow \quad F = mv\frac{dv}{dx}$$

$$\Rightarrow \quad Fdx = mvdv$$

$$\Rightarrow \quad \int Fdx = \int mvdv$$

considering the work done when the particle has:
original velocity v_1 at displacement x_1
final velocity v_2 at displacement x_2

$$\Rightarrow \quad \int_{x_1}^{x_2} Fdx = \int_{v_1}^{v_2} mvdv$$

$$F\left[x\right]_{x_1}^{x_2} = \left[\tfrac{1}{2}mv^2\right]_{v_1}^{v_2}$$

$$\underline{F(x_2 - x_1) = \tfrac{1}{2}mv_2^2 + \tfrac{1}{2}mv_1^2}$$

Gravitational Potential Energy

This is the energy a mass posesses by virtue of its position. It is equal to the product of mass, gravitational field strength(g) and the vertical distance the particle is above a fixed level.

m = mass speed(ms^{-1}), g = gravitational field strength(N/kg)

h = vertical distance(m)

potential energy P.E.(joules) = mgh

(note the unit for g - the gravitational force on a mass of 1kg)

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The Law of Conservation of Energy

In a closed system the amount of energy is constant. Or in other words 'energy can never be created nor destroyed', it mearly changes from one form into another.

This is the classical physics view that is useful for most purposes. However, in the real world systems are seldom perfect. We also have the problem when referring to particle physics that energy can indeed be created and destroyed. Annihilation of elementary particles is an example of this(matter-antimatter: electron-positron collision).

Example #1

A pump forces up water at $8ms^{-1}$ from a well into a reservoir at a rate of 50 kg s^{-1} .
If the water is raised a vertical height of 40 m, what is the work done per second?(assume g=10 ms^{-2})

$m = 50\text{kg} \quad h = 40\text{m} \quad g = 10\text{ms}^{-2} \quad v = 8\text{ms}^{-1}$

$$\begin{aligned}\text{work done} &= \text{increase in PE} + \text{increase in KE}\\ &= mgh + \tfrac{1}{2}mv^2\\ &= (50\times10\times40) + (0.5\times50\times8\times8)\\ &= 20,000 + 1,600 = 21,600\end{aligned}$$

Ans. work done per second is 21,600J.

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Example #2

A gun is fireed at a 3 cm thick solid wooden door.
The 7g bullet travels through the door and speed reduces from 450 ms^{-1} to 175 ms^{-1}.
Assuming uniform resistance, what is the force of the wood on the bullet. (5 sig. figs.)

$u = 450\text{ms}^{-1} \quad v = 175\text{ms}^{-1} \quad s = 3\text{cm} \equiv 0.03\text{m}$
$m = 7\text{g} \equiv 0.007\text{kg}$

using $v^2 - u^2 = 2as$

$$a = \frac{v^2 - u^2}{2s}$$
$$= \frac{(175)^2 - (450)^2}{2 \times 0.03}$$
$$= \frac{30625 - 202500}{0.06}$$
$$= 3885416.667$$

using $F = ma$,

$$F = 0.007 \times 3885416.667$$
$$= 27197.917$$

Ans. resistive force from wood is 27198N (5 sig.figs.)

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Example #3

In a science experiment, a 50g mass slides down a 60° incline of length 0.5m.
If the mass is given an initial speed of 2 ms^{-1} down the plane and its final speed is measured as 3 ms^{-1}, what is the magnitude of the frictional force opposing the mass?
(assume g=10 ms^{-2} , answer to 2d.p.)

$m = 50\text{g} \equiv 0.05\text{kg} \quad s = 0.5\text{m} \quad u = 2\text{ms}^{-1} \quad v = 3\text{ms}^{-1}$
f frictional force

energy at start = energy at end

$$\text{PE}_{\text{start}} + \text{KE}_{\text{start}} = \text{PE}_{\text{end}} + \text{KE}_{\text{end}} + \text{work done}$$

$$mgh + \tfrac{1}{2}mu^2 = 0 + \tfrac{1}{2}mv^2 + fS$$

$$(0.05\times10\times0.433) + (0.5\times0.05\times4) = (0.5\times0.05\times9) + 0.5f$$

$$0.2165 + 0.1 = 0.225 + 0.5f$$

$$0.3165 - 0.225 = 0.5f$$

$$f = \frac{0.915}{0.5} = 1.83$$

Ans. frictional force opposing mass movement is 1.83N

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Kinetics: Power & Efficiency

Power

Power is by definition the rate of working.
Since **work = force x distance moved**, it follows that :

$$\text{power} = \frac{\text{force} \times \text{distance moved}}{\text{time}}$$
$$\equiv \frac{\text{Newtons} \times \text{metres}}{\text{seconds}}$$
$$\equiv \text{Nms}^{-1}$$

1 joule is the work done when 1 newton moves its point of application through 1 metre,

1 joule = 1 newton x 1 metre

1J = 1 Nm

1 watt is a rate of working of 1 joule per second

$1W = 1\ Js^{-1}$

Example

A military tank of mass 20 metric tonnes moves up a 30° hill at a uniform speed of 5 ms^{-1} . If all the frictional forces opposing motion total 5000N, what is the power delivered by the engine?
(g = 10ms^{-2} , answer in kW)

If the tank is moving at constant speed then the forces forwards are balanced by the forces backwards.

m is the tank's mass, then mgsin30° is the component of the weight down the hill
R is the total of resistive forces down the hill
T is the tractive force forwards up the hill

$$mg\sin30^\circ + R = T$$

$$T = (20{,}000 \times 10 \times 0.5) + 5000 = 105{,}000N$$

power = force x speed

power of tank engine = 105,000 x 5 = 525,000W

Ans. 525 kW

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Efficiency

Efficiency is the ratio of useful work out divided by total work done, expressed as a percentage.

$$\text{efficiency}(\%) = \frac{\text{useful work out}}{\text{total work done}} \times 100$$

Example

A pump running at an efficiency of 70% delivers oil at a rate of 4 kgs^{-1} with a speed of 3 $ms^{-1}$to an oil heater .
if the vertical distance moved by the oil is 10 m, what is the power consumption of the pump?
(g = 10 ms^{-2}, answer to 1 d.p.)

E_f = 70%, m=4kg, v=3 ms^{-1} , h=10 m, g=10 ms^{-2}

work/sec. to raise oil 8 m high = mgh = 4x10x10 = 400 J/s
work/sec. to produce discharge speed = 0.5x4x3x3 = 18 J/s
total work/sec. = 400 + 18 = 418 W

418 W represents 70% of the power supplied

therefore total power consumption of pump =

$$\frac{418}{70} \times \frac{100}{1} = \frac{41800}{70} = 597.143$$

Ans. power consumption of pump is 597.1W (1.d.p.)

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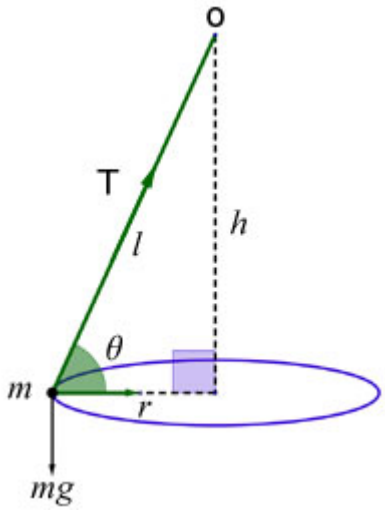
Kinetics: More Circlular Motion

Conical pendulum

Problems concerning the conical pendulum assume no air resistance and that the string has no mass and cannot be stretched.

Solution of problems involves resolving forces on the mass vertically and horizontally. In this way the speed of the mass, the tension in the string and the period of revolution can be ascertained.



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resolving forces vertically on the mass

$$mg = T\sin\theta$$

$$T = \frac{mg}{\sin\theta} \qquad \text{(i}$$

resolving forces horizontally on the mass

$$T\cos\theta = \frac{mv^2}{r}$$

$$T = \frac{mv^2}{r\cos\theta} \qquad \text{(ii}$$

eliminating T by combining (i and (ii

$$\frac{mg}{\sin\theta} = \frac{mv^2}{r\cos\theta}$$

$$gr = v^2\tan\theta$$

$$v = \sqrt{\frac{gr}{\tan\theta}}$$

but $\tan\theta = \frac{h}{r}$

$$\Rightarrow \qquad v = \sqrt{gr\left(\frac{r}{h}\right)}$$

$$\Rightarrow \qquad v = \sqrt{\left(\frac{gr^2}{h}\right)}$$

$$\Rightarrow \qquad \underline{v = r\sqrt{\left(\frac{g}{h}\right)}}$$

$$\text{the period of revolution} = \frac{\text{circumference}}{\text{speed}}$$

$$= \frac{2\pi r}{v}$$

substituting for v using $v = r\sqrt{\frac{g}{h}}$

$$\text{the period of revolution} = \frac{2\pi r}{r\sqrt{\frac{g}{h}}} = \frac{2\pi r}{r}\sqrt{\frac{h}{g}}$$

$$\underline{\text{the period of revolution} = 2\pi\sqrt{\frac{h}{g}}}$$

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Example

A 20g mass moves as a conical pendulum with string length $8x$ and speed v.
if the radius of the circular motion is $5x$ find:
i) the string tension(assume g =10 ms^{-2} , ans. to 2 d.p.)
ii) v in terms of x, g

i)

$$l = 8x \quad r = 5x \quad \cos^{-1}\theta = \tfrac{5}{8} \quad \theta = 51.3^\circ$$
$$m = 20\text{g} \equiv 0.02\text{kg} \quad g = 10\text{ms}^{-2}$$

resolving vertically

$$T\sin\theta = mg$$
$$T = \frac{mg}{\sin\theta}$$
$$= \frac{0.02\times 10}{0.7804} = 0.2563$$

Ans. the string tension T is 0.26N (2.d.p.)

ii)

resolving horizontally

$$T\cos\theta = \frac{mv^2}{r}$$

substituting for T, from $T = \frac{mg}{\sin\theta}$ above

$$mg\frac{\cos\theta}{\sin\theta} = \frac{mv^2}{r}$$
$$\frac{gr}{\tan\theta} = v^2$$
$$v = \sqrt{\frac{gr}{\tan\theta}}$$

substituting for $r = 5x$, $\theta = 51.3^\circ$

$$v = \sqrt{\frac{5gx}{\tan(51.3^\circ)}} = \sqrt{\frac{5}{\tan(51.3^\circ)}}\cdot\sqrt{gx} = 2\sqrt{gx}$$

Ans. velocity v in terms of g, x is $2\sqrt{gx}$

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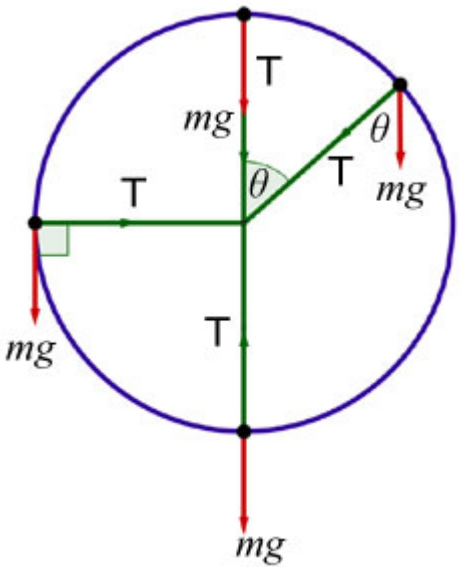
Mass performing vertical circular motion under gravity



Consider a mass m performing circular motion under gravity, the circle with radius r .
The centripetal force on the mass varies at different positions on the circle.

top $\quad mg + T = \frac{mv^2}{r}$

middle $\quad T = \frac{mv^2}{r}$

bottom $\quad T - mg = \frac{mv^2}{r}$

string at an angle θ to the vertical

$$mg\cos\theta + T = \frac{mv^2}{r}$$

For many problems concerning vertical circular motion, energy considerations(KE & PE) of particles at different positions are used to form a solution.

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Example #1

A 50g mass suspended at the end of a light inextensible string performs vertical motion of radius 2m.
If the mass has a speed of 5 ms^{-1} when the string makes an angle of 30° with the vertical, what is the tension? (assume g =10 ms^{-2} , answer to 1 d.p.)

$m = 50\text{g} \equiv 0.05\text{kg} \quad v = 5\text{ms}^{-1} \quad \theta = 30^\circ \quad r = 2\text{m}$

$g = 10\text{ms}^{-2}$

the centripetal force is the sum of the tension in the string and the component of the weight along the string

$$\Rightarrow \quad mg\cos\theta + T = \frac{mv^2}{r}$$

$$\Rightarrow \quad T = \frac{mv^2}{r} - mg\cos\theta$$

$$= \frac{(0.05)(5)^2}{2} - (0.05)(10)\cos 30^\circ$$

$$= 0.625 - 0.433 = 0.192$$

Ans. tension in string is 0.2N

Example #2

A 5kg mass performs circular motion at the end of a light inextensible string of length 3m. If the speed of the mass is 2 ms^{-1} when the string is horizontal, what is its speed at the bottom of the circle? (assume g =10 ms^{-2})

$v_H = 2\text{ms}^{-1} \quad r = 3\text{m} \quad g = 10\text{ms}^{-1}$

v_B speed at bottom of circle

PE is measured relative to the bottom of the circle

KE + PE string horizontal = KE + PE at bottom

$$\tfrac{1}{2}mv_H^2 + mgr = \tfrac{1}{2}mv_B^2 + 0$$

$$v_H^2 + 2gr = v_B^2$$

$$v_B = \sqrt{v_H^2 + 2gr}$$

$$= \sqrt{(2)^2 + 2\times(10)\times(3)}$$

$$= \sqrt{4 + 60} = \sqrt{64} = 8$$

Ans. speed at bottom of circle is 8 ms^{-1}

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Kinetics: Elastic Strings

Hooke's Law

The extension e of an elastic string/spring is proportional to the extending force F.

$$F \propto e$$
$$F = ke$$

where k is the constant of proportionality, called the spring constant.

By definition the **spring constant** is that force which will produce unit extension(unit Nm^{-1}) in a spring.

example

A 2kg mass hangs at the end of an elastic string of original length 0.5 m.
If the spring constant is 60 Nm^{-1}, what is the length of the extended elastic string?

$m = 2\text{kg} \quad k = 60\text{Nkg}^{-1} \quad l_o = 0.5\text{m}$
$e = l - l_o \quad g = 10\text{ms}^{-2}$

resolving forces vertically on mass

$$mg = ke$$

substituting for $e = l - l_o$

$$mg = k(l - l_o)$$
$$\frac{mg}{k} = (l - l_o)$$
$$l = l_o + \frac{mg}{k}$$
$$l = 0.5 + \left(\frac{2 \times 10}{60}\right) = 0.8\dot{3}$$

Ans. length of extended string is 0.8 m(1 d.p.)

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The energy stored in a stretched string/spring
(same as the work done in stretching a string)

Remembering that 'work = force x distance moved', if the elastic string is extended a small amount δe(such that the force F is considered constant) then the work done δW is given by:

$$\delta W = F\,\delta e \qquad \text{(i}$$

substituting for $F = ke$ in (i

$$\Rightarrow \qquad \delta W = ke\,\delta e$$

work W done in extension 0 to e

$$W = \int_0^e ke\,de$$

$$= k\int_0^e e\,de$$

$$= k\left[\frac{e^2}{2}\right]_0^e$$

$$= \frac{ke^2}{2}$$

$$\underline{W = \tfrac{1}{2}ke^2}$$

example

A 2 kg mass m hangs at a point B at the end of an elastic string of natural length 0.7 m supported at a point A.
The extension produced in the string by the mass is a fifth of the original length, while the spring constant is $5mg$.
If the mass is now held at point A and then released, what is the maximum speed the mass will attain?
(g = 10 ms^{-2} , answer to 1 d.p.)

Let PE be related to the equilibrium position of the mass. Hence the mass will have maximum KE when passing through point B. Below that point there will be no net force downwards to produce acceleration. The net force will act in the opposite direction, producing deceleration.

In the diagram,

a_1 is linear acceleration under gravity

a_2 is non-linear acceleration(decreasing)

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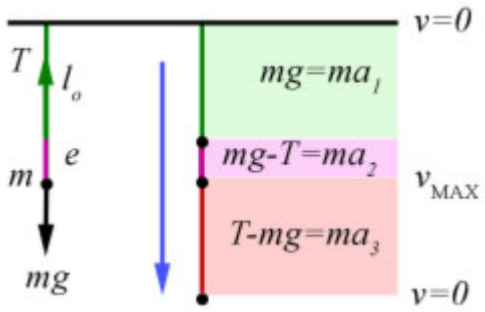
a_3 is negative acceleration i.e. deceleration.



$m = 2\text{kg} \quad l_o = 0.7\text{m} \quad e = \frac{l_o}{5} \quad l = \frac{6}{5}\ l_o$

$g = 10\text{ms}^{-2} \quad k = 5mg$

PE at A = (KE at B) + (elastic PE at B)

$$mgl = \frac{1}{2}mv^2 + \frac{1}{2}ke^2$$

$$\frac{6}{5}mgl_o = \frac{1}{2}mv^2 + \frac{5}{2}mg\left(\frac{l_o}{5}\right)^2$$

$$\frac{6}{5}gl_o = \frac{v^2}{2} + \frac{gl_o^2}{10}$$

multiplying both sides by 10

$$12gl_o = 5v^2 + gl_o^2$$

$$5v^2 = 12gl_o + gl_o^2$$

$$v = \sqrt{\frac{12gl_o + gl_o^2}{5}}$$

$$= \sqrt{\frac{(12\times10\times0.7) + (10\times0.7\times0.7)}{5}}$$

$$= 4.2166$$

Ans. maximum speed of mass is 4.2ms^{-1} (1d.p.)

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Momentum & Impulse: Impulse

Momentum

Momentum is by definition the product of mass and velocity. So strictly speaking momentum is a vector quantity.

$$\text{momentum} = \text{mass(kg)} \times \text{velocity}(ms^{-1})$$

Hence the unit of momentum is $(kg.ms^{-1})$.

Impulse of a force

This is simply the force multiplied by the time the force acts.

We can obtain an expression for this in terms of momentum from Newton's Second Law equation $F=ma$, where the force F is constant.

Remembering that velocity, force and therefore impulse are vector quantities.

For a mass *m* being accelerated by a constant force **F**, where the impulse is **J** , $\mathbf{v}_1$ is initial velocity and $\mathbf{v}_2$ is final velocity:

$$\mathbf{F}t = m(\mathbf{v}_2 - \mathbf{v}_1)$$

$$\mathbf{J} = \mathbf{F}t$$

$$\mathbf{J} = m(\mathbf{v}_2 - \mathbf{v}_1)$$

Units

Since impulse is the product of force and time, the units of impulse are (Newtons) x (seconds), or N s .

Vector problems

Vector type questions on impulse are solved by first calculating the change in momentum. This gives a vector expression for the impulse. Using Pythagoras, the magnitude of the impulse can then be found. The anglular direction is calculated from the coefficients of unit vectors **i** and **j**.

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Example #1

A particle of mass 0.5 kg moves with a constant velocity of $(3\mathbf{i} + 5\mathbf{j})$ m.s^{-1} .
After being given an impulse, the particle then moves off with a constant velocity of $(2\mathbf{i} - 3\mathbf{j})$ m.s^{-1} .

Calculate:

i) the impulse
ii) the magnitude of the impulse(to 2 d.p.)
iii) the direction of the impulse(θ degrees to the x-axis)

i)

$\mathbf{v}_1 = (3\mathbf{i} + 5\mathbf{j})$ $\mathbf{v}_2 = (2\mathbf{i} - 3\mathbf{j})$ $m = 0.5$ kg

using $\mathbf{J} = m(\mathbf{v}_2 - \mathbf{v}_1)$

$\mathbf{J} = 0.5(2\mathbf{i} - 3\mathbf{j}) - 0.5(3\mathbf{i} + 5\mathbf{j})$

$\mathbf{J} = \mathbf{i} - 1.5\mathbf{j} - 1.5\mathbf{i} - 2.5\mathbf{j}$

$\mathbf{J} = (1 - 1.5)\mathbf{i} + (-1.5 - 2.5)\mathbf{j}$

$\underline{\mathbf{J} = (0.5\mathbf{i} - 4\mathbf{j})}$ N.s

ii)

magnitude of impulse = $\sqrt{[(0.5)^2 + (-4)^2]} = \sqrt{[16.23]}$

= 4.03 N.s

iii) direction $\tan^{-1}\theta = (4)/(0.5) = 8$

$\theta = 82.8749^o = \underline{82.87^o}$ (2 d.p.) clockwise to the x-axis

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Example #2

A particle of mass 2.5 kg is moving with a constant velocity of $(2\mathbf{i} + \mathbf{j})$ m.s^{-1} .
After an impulse, the particle moves with a constant velocity of $(4\mathbf{i} + 3\mathbf{j})$ m.s^{-1} .

Calculate:

i) the impulse
ii) the magnitude of the impulse(to 2 d.p.)
iii) the angle(θ^o) the impulse makes with the x-axis

$\mathbf{v}_1 = (2\mathbf{i} + \mathbf{j})$ $\mathbf{v}_2 = (4\mathbf{i} + 3\mathbf{j})$ $m = 2.5$ kg

using $\mathbf{J} = m(\mathbf{v}_2 - \mathbf{v}_1)$

$\mathbf{J} = 2.5(4\mathbf{i} + 3\mathbf{j}) - 2.5(2\mathbf{i} + \mathbf{j})$

$\mathbf{J} = 10\mathbf{i} + 7.5\mathbf{j} - 5\mathbf{i} - 2.5\mathbf{j}$

$\mathbf{J} = (10 - 5)\mathbf{i} + (7.5 - 2.5)\mathbf{j}$

$\underline{\mathbf{J} = (5\mathbf{i} + 5\mathbf{j})}$

ii)

magnitude of impulse = $\sqrt{[(5)^2 + (5)^2]} = \sqrt{[50]} = \underline{7.07 \text{ N.s}}$

iii) direction $\tan^{-1}\theta = (5)/(5) = 1$

$\theta = \underline{45^o}$ $\underline{\text{anticlockwise to the x-axis}}$

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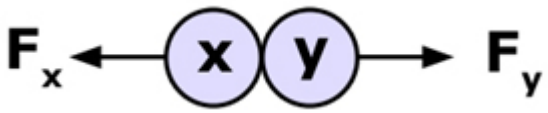
Momentum & Impulse: Conservation of Momentum

The Principle of Conservation of Momentum

The **total linear momentum** of a system of colliding bodies, with no external forces acting, **remains constant**.

for two perfectly elastic colliding bodies note:

i) By Newton's 3rd. Law, the force on X due to Y , (F_x) is the same as the force on Y due to X , (F_y) .



$F_x = F_y$

ii) By Newton's 2nd. Law, the rate of change of momentum is the same, since F = (rate of change of momentum)

iii)Because the directions of the momentum of the objects are opposite, (and therefore of different sign) the net change in momentum is zero.

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Example #1

A 5 kg mass moves at a speed of 3 ms^{-1} when it collides head on, with a 3 kg mass travelling at 4 ms^{-1}, travelling along the same line.
After the collision, the two masses move off together with a common speed.

What is the common speed of the combined masses?

$m_1 = 5$ kg $\quad v_1 = 5\ ms^{-1} \qquad m_2 = 3$ kg $\qquad v_2 = -4\ ms^{-1}$

let the combined speed after collision be v_3

then, by the law of conservation of momentum,

momentum before collision = momentum after collision

$$m_1v_1 + m_2v_2 = (m_1 + m_2)v_3$$

$$\Rightarrow \qquad v_3 = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}$$

substituting for m_1 v_1 m_2 and v_2

$$v_3 = \frac{(5 \times 3) + (3 \times (-4))}{5 + 3}$$

$$= \frac{15 - 12}{8} = \frac{3}{8} = 0.375$$

common speed of the two masses is 0.375 ms^{-1}

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Example #2

An artillery shell of mass 10 kg is fired from a field gun of mass 2000 kg.
If the speed of the shell on leaving the muzzle of the gun is 250 ms^{-1} , what is the recoil speed of the gun?

$m_1 = 10$ kg $\quad v_1 = 250\ ms^{-1} \quad m_2 = 2000$ kg
let the recoil speed of the gun after firing be v_2

then, by the law of conservation of momentum,

momentum before firing = momentum after firing

$$0 = m_1v_1 + m_2v_2$$

$$\Rightarrow \quad -m_2v_2 = m_1v_1$$

$$\Rightarrow \quad v_2 = \frac{m_1v_1}{-m_2}$$

substituting for m_1 v_1 and m_2

$$v_2 = \frac{10 \times 250}{-2000} = -\frac{25}{20} = -1.25$$

(the minus signifies gun moves in opposite direction to the shell)

the speed of recoil of the gun is $1.25\ ms^{-1}$

Energy changes during collisions

Consider the kinetic energy change involved during a collision. Remember that no energy is actually lost, it is just converted into other forms. Energy can be transformed into heat, sound and permanent material distortion. The later causes the internal potential energy of bodies to increase.

If no kinetic energy is lost (K.E.= ½ mv^2) then the collision is said to be **perfectly elastic**. However if kinetic energy is lost, the collision is described as **inelastic**. In the special case when all the kinetic energy is lost, the collision is described as **completely inelastic**. This is when to two colliding bodies stick to one another on impact and have zero combined velocity.

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Momentum & Impulse: Coefficient of Restitution

The Coefficient of Restitution

The **Coefficient of Restitution (e)** is a variable number with no units, with limits from zero to one.

$$0 \leq e \leq 1$$

'e' is a consequence of **Newton's Experimental Law of Impact**, which describes how the speed of separation of two impacting bodies compares with their speed of approach.

note: the speeds are **relative** speeds

v_a = speed of approach v_s = speed of separation

e = coefficient of restitution

$$\text{coefficient of restitution} = \frac{\text{speed of separation}}{\text{speed of approach}}$$

$$e = \frac{v_s}{v_a}$$

If we consider the speed of individual masses before and after collision, we obtain another useful equation:

u_A = initial speed of mass A

u_B = initial speed of mass B

v_A = final speed of mass A

v_B = final speed of mass B

relative initial speed of mass A to mass B = $u_B - u_A$

relative final speed of mass A to mass B = $v_B - v_A$

$$\text{coefficient of restitution (e)} = \frac{v_B - v_A}{u_B - u_A}$$

note*:* in this equation the absolute of $u_B - u_A$ and $v_B - v_A$ are used (|absolute| no net negative result)

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Example

A 5 kg mass moving at 6 ms^{-1} makes a head-on collision with a 4 kg mass travelling at 3 ms^{-1} .
Assuming that there are no external forces acting on the system, what are the velocities of the two masses after impact?
(assume coefficient of restitution e = 0.5)

u_A = initial speed of 5 kg mass (mass A)= 6 ms^{-1}

u_B = initial speed of 4 kg mass (mass B)= 3 ms^{-1}

m_A = 5 kg m_B = 4 kg

v_A = final speed of mass A v_B = final speed of mass B

momentum before the collision equals momentum after

hence, $m_A u_A + m_B u_B = m_A v_A + m_B v_B$

also

$$\text{coefficient of restitution (e)} = \frac{v_B - v_A}{u_B - u_A}$$

substituting for e, m_A , u_A , m_B , u_B

we obtain two simultaneous equations

from the conservation of momentum,

$$5 \times 0.6 + 4(-3) = 5 v_A + 4 v_B$$

$$3 - 12 = 5 v_A + 4 v_B$$

$$-9 = 5 v_A + 4 v_B$$

$$5 v_A + 4 v_B = -9 \qquad \text{(i}$$

from the coefficient of restitution expression,

$$0.5 = \frac{v_B - v_A}{u_B - u_A}$$

$$0.5(u_A - u_B) = v_B - v_A$$

$$(0.5 \times 6) - (0.5(-3)) = v_B - v_A$$

$$3 + 1.5 = v_B - v_A$$

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$$4.5 = v_B - v_A$$

$$v_B - v_A = 4.5 \qquad \text{(ii}$$

multiplying (ii by 5 and adding

$$5\ v_A + 4\ v_B = -9$$

$$-5\ v_A + 5\ v_B = 22.5$$

$$9\ v_B = 13.5$$

$$\boldsymbol{v_B = 1.5\ ms^{-1}}$$

from (ii

$$1.5 - v_A = 4.5$$

$$v_A = 1.5 - 4.5 = -3$$

$$\boldsymbol{v_A = -3\ ms^{-1}}$$

Ans. The velocities of the 5 kg and 4 kg masses are -3 $ms^{-1}$and 1.5 ms^{-1}, respectively.

Oblique Collisions

For two masses colliding along a line, Newton's Experimental law is true for component speeds. That is, the law is applied twice: to each pair of component speeds acting in a particular direction.

Example

A particle of mass *m* impacts a smooth wall at $4u$ ms^{-1} at an angle of 30 deg. to the vertical. The particle rebounds with a speed *ku* at 90 deg. to the original direction and in the same plane as the impact trajectory.

What is:
i) the value of the constant '*k*' ?
ii) the coefficient of restitution between the wall and the particle?
iii) the magnitude of the impulse of the wall on the particle

i) There is no momentum change parallel to the wall.

$$m(4u)\cos 30^\circ = m(ku)\cos 60^\circ$$

$$4u\frac{\sqrt{3}}{2} = ku.\frac{1}{2}$$

$$4\sqrt{3} = k, \qquad \underline{k = 4\sqrt{3}}$$

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ii) The coefficient of restitution 'e' is the ratio of the speed of separation to the speed of approach:

$$e = \frac{\text{speed of separation}}{\text{speed of approach}} = \frac{ku\sin 60^\circ}{4u\sin 30^\circ}$$

$$= \frac{4\sqrt{3}.\frac{\sqrt{3}}{2}u}{2u} = \frac{6u}{2u} = 3$$

$$\underline{e = 3}$$

iii) The impulse is the change of momentum.

Since the vertical unit vectors are unchanged, the momentum change just concerns the horizontal vector components.

hence,

$$\text{momentum change} = m(4u\sin 30^\circ) - m(-ku\sin 60^\circ)$$

$$= m.4u.\frac{1}{2} + m.4\sqrt{3}u\frac{\sqrt{3}}{2}$$

$$= 2mu + 6mu$$

$$= 8mu$$

$$\underline{\text{ans. momentum change is } 8mu \text{ N.s}}$$

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Statics: Particle Forces in Equilibrium

Below is a list of methods for describing forces in equilibrium acting on a particle. The forces act in **one** plane and are called **coplanar** .
In this context the word **equilibrium** means that the forces are in balance and there is no net force acting.

Vector Method

The condition for equilibrium is that the vector sum of forces(the resultant)is the **null vector**. This means that the multipliers for the **i**, **j** and **k** unit vectors are each zero.

Example #1

A particle at rest has forces (2**i** + 3**j**), (*m***i** + 6**j**) and (-4**i** + *n***j**) acting on it.
What are the values of *m* and *n* ?

For equilibrium, all the forces when added together(the vector sum) equal zero.

$$2\boldsymbol{i}+3\boldsymbol{j}+m\boldsymbol{i}+6\boldsymbol{j}-4\boldsymbol{i}+n\boldsymbol{j}=0$$
$$(2+m-4)\boldsymbol{i}+(3+6+n)\boldsymbol{j}=0$$

The scalar multipliers of each unit vector equal zero.

$$2+m-4=0$$
$$m=4-2$$
$$\underline{m=2}$$

$$3+6+n=0$$
$$n=-6-3$$
$$\underline{n=-9}$$

Ans. $m = 2$ and $n = -9$

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Example #2

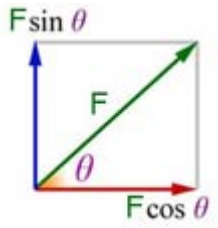
A particle at rest has an unknown force F and two other forces (2**i** - 5 **j**), (-3**i** + **j**), acting on it.
What is the magnitude of F ? (2 d.p.)
(all forces in Newtons, N)

for equilibrium the vector sum equals zero

$$2\boldsymbol{i} - 5\boldsymbol{j} - 3\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{F} = 0$$
$$\boldsymbol{F} = (2-3)\boldsymbol{i} + (-5+1)\boldsymbol{j}$$
$$\boldsymbol{F} = -\boldsymbol{i} - 4\boldsymbol{j}$$
$$|\boldsymbol{F}| = \sqrt{(-1)^2 + (-4)^2} = \sqrt{1+16} = \sqrt{17} = 4.12$$

Ans. force F = 4.12 N

Resolution Method



A force **F** can be replaced by two vectors that are at right angles to each other, passing through the point of application. Hence if the angle between one component vector and the original vector is θ(theta), then the two components are **F**sinθ and **F**cosθ .

Problems are solved by resolving all the vectors into their horizontal and vertical components. The components are then resolved vertically and then horizontally to obtain two equations. These can be solved as simultaneous equations.

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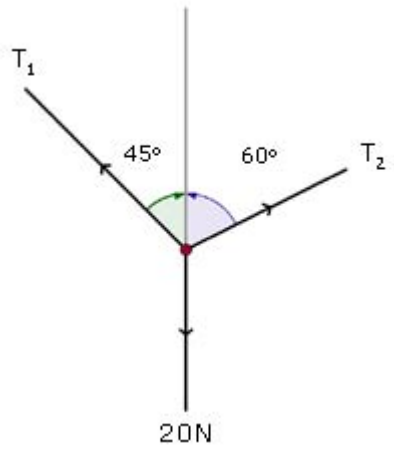
Example

A 2 kg mass is suspended by two light inextensible strings inclined at 60 deg. and 45 deg. to the vertical.
What are the tensions in the strings?(to 2 d.p.)
(assume g=10 ms^{-2})



resolving horizontally $T_1 \sin 60^\circ = T_2 \sin 45^\circ$

$$T_1 = T_2 \frac{\sin 45^\circ}{\sin 60^\circ} = \frac{0.7071}{0.8660} T_2 = 0.8165 T_2$$

$$T_1 = 0.8165 T_2 \qquad \text{(i}$$

resolving vertically

$$T_1 \cos 60^\circ + T_2 \cos 45^\circ = 20 \qquad \text{(ii}$$

substituting for T_1 in (ii

$$0.8165 T_2 (0.5) + 0.7071 T_2 = 20$$

$$0.4083 T + 0.7071 T_2 = 20$$

$$1.1154 T_2 = 20$$

$$T_2 = \frac{20}{1.1154} = 17.9308$$

from (i $T_1 = 0.8165 T_2$

$$T_1 = 0.8165 \times 17.9308 = 14.6405$$

Ans. the tensions in the strings are 14.6 N and 17.9 N

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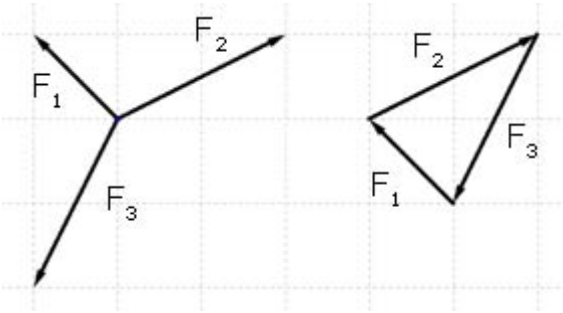
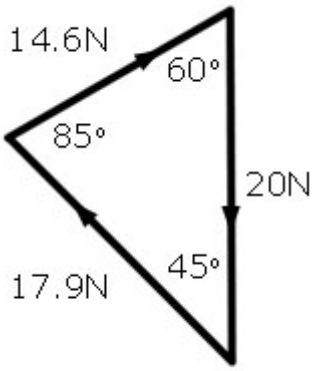
Triangle of Forces

When 3 coplanar forces acting at a point are in equilibrium, they can be represented in magnitude and direction by the adjacent sides of a triangle taken in order.



Example

Using the results from the previous example, the three forces acting on the 2 kg mass can be represented by a scale diagram.



Our starting point is the 20N force acting downwards. One force acts at 45 deg. to this line and the other at 60 deg. So to find the magnitude of the two forces, draw lines at these angles at each end of the 20N force. Where the lines cross gives a vertex of the triangle. Measuring the lengths of the lines from this to the ends of the 20N force line will give the magnitudes of the required forces.

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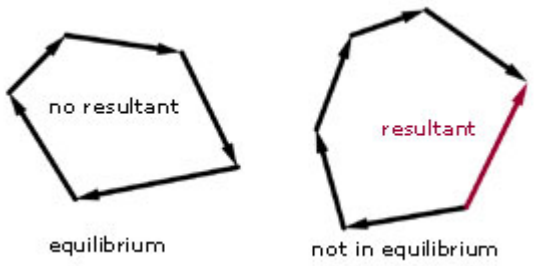
Polygon of Forces

For equilibrium, forces are represented in magnitude and direct to form a polygon shape.

If a number of forces are acting at a point, then the missing side in the polygon represents the resultant force. Note the arrow direction on this force is in the opposite direction to the rest.



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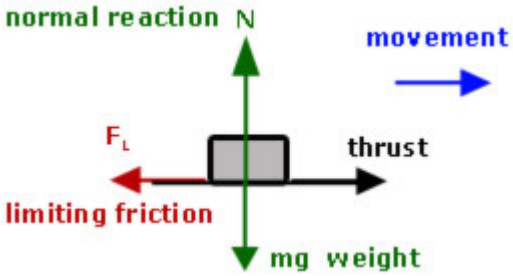
Statics: Friction

Friction is the force that opposes movement.



The 3 Laws of Friction

1.

The **limiting frictional force** ($\mathbf{F_L}$)is directly proportional to the **normal contact force** (**N**)

$$F_L \propto N$$

note: limiting frictional force is the maximum frictional force

2.

The ratio of the **limiting frictional force** ($\mathbf{F_L}$) to the **normal contact force** (**N**) is called the coefficient of friction (μ)

$$\mu = \frac{F_L}{N} \qquad F_L = \mu N$$

3.

When there is no motion, but the object is on the point of moving,

applied force = frictional force(limiting friction)

and when there is motion,

applied force > frictional force(limiting friction)

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then this equality applies:

$$F_L = \mu N$$

Up to this point, when the frictional force is **less** than limiting friction(maximum)*, then the inequality below applies.

$$F_L < \mu N$$

*object is static and not on the point of moving

Example #1

A flat stone is thrown horizontally across a frozen lake.
If the stone decelerates at 2.5 ms^{-2} , what is the coefficient of friction between the stone and the ice? (take g=10 ms^{-2})

since the stone is moving, the friction is limiting(max.)

$\therefore \quad F_L = \mu N$ (i

and vertically, $N = mg$

the stone is decelerating,

using Newton's Second law,

$$F_L = ma$$

substituting for F_L and N into (i

$$ma = \mu mg$$

$$\Rightarrow \quad \mu = \frac{ma}{mg} = \frac{a}{g} = \frac{2.5}{10} = 0.25$$

Ans. coefficient of friction between stone & ice is 0.25

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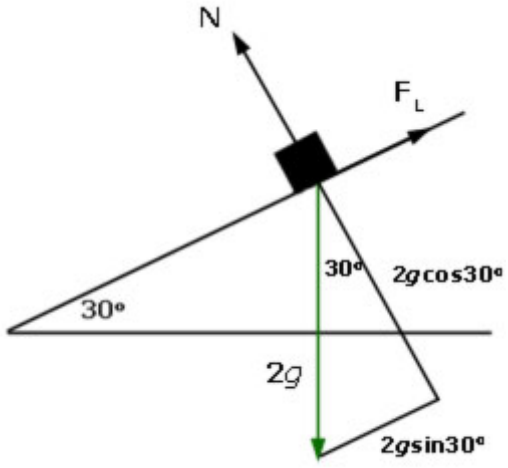
Example #2

A 2 kg mass in limiting equilibrium rests on a rough plane inclined at an angle of 30 deg. to the horizontal.
Show that the coefficient of friction between the mass and the plane is √3 / 3 .



resolving along the plane

$$F_L = 2g\sin 30^\circ = 2g.\frac{1}{2}$$

$$F_L = g$$

resolving at right angles to the plane

$$N = 2g\cos 30^\circ$$

$$N = 2g\left(\frac{\sqrt{3}}{2}\right) = g\sqrt{3}$$

for limiting friction $F_L = \mu N$

$$\Rightarrow \qquad \mu = \frac{F_L}{N} = \frac{g}{g\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

Ans. coefft. of friction between 2 kg mass & plane is $\frac{\sqrt{3}}{3}$

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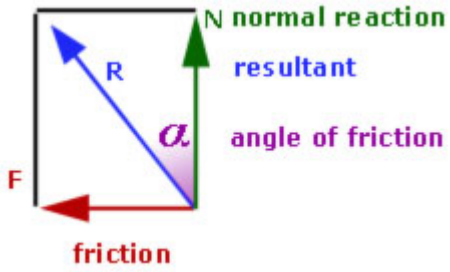
The Angle of Friction



If we examine the normal reaction force(***N***) and the frictional force(F_L) when it is limiting, then the equation $F_L = \mu N$ applies.

If the resultant between ***N*** and F_L is ***R*** , and it is inclined at an angle α(alpha) to the normal ***N***, then we can write equations for F_L and ***N*** in terms of ***R*** .

$$\frac{F_L}{N} = \mu$$

$$F_L = R\sin\alpha$$
$$N = R\cos\alpha$$
$$\frac{F_L}{N} = \frac{R\sin\alpha}{R\cos\alpha} = \tan\alpha$$

$$\Rightarrow \quad \underline{\mu = \tan\alpha}$$

In example #2(above) the angle of friction = $\tan^{-1}(\sqrt{3} / 3)$
= $\tan^{-1}(0.5773) = 30^o$ (the angle of the plane)

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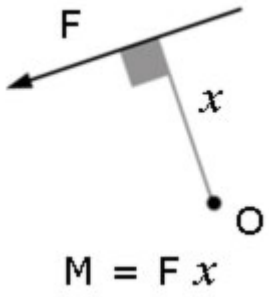
Statics: Rigid Bodies

The Moment of a Force

The moment M (turning effect) of a force about a point O is the product of the magnitude of the force (F) and the perp. distance (x)to the point of application.



By convention, **anti-clockwise** moments are **positive**.

The Principle of Moments

For a rigid body acted upon by a system of coplanar forces, equilibrium is achieved when:

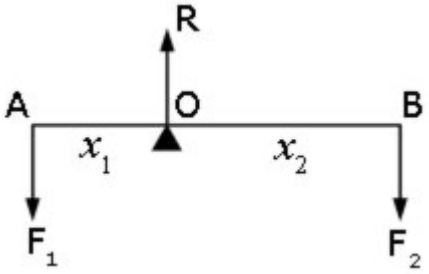
i) the vector sum of the coplanar forces = 0

ii) there is no net turning effect produced by the forces
(the sum of clockwise & anti-clockwise moments = 0)

Parallel forces acting on a beam

When attempting problems concerning a balance points or fulcrum, remember that there is always an upward force acting.

Example #1 fulcrum near centre



Consider two forces F_1 and F_2 acting vertically downwards at either end of a beam of negligible mass.

When the beam is balanced at its fulcrum O(i.e. horizontal), the sum of the downward forces equals the sum of the upward forces.

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If the reaction force at the fulcrum is *R*,

$$R = F_1 + F_2$$

taking moments about O,

$$F_1 x_1 - F_2 x_2 = 0$$

$$F_1 x_1 = F_2 x_2$$

$$\frac{F_1}{F_2} = \frac{x_2}{x_1}$$

taking moments about A,

$$(F_1 + F_2) x_1 - F_2 (x_1 + x_2) = 0$$

$$(F_1 + F_2) x_1 = F_2 (x_1 + x_2)$$

$$\underline{\frac{x_1}{x_1 + x_2} = \frac{F_2}{F_1 + F_2}}$$

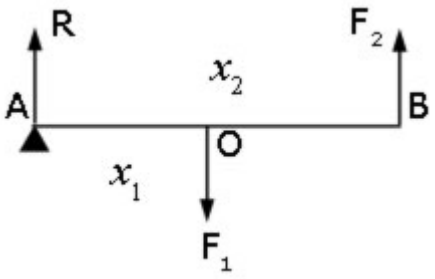
taking moments about B,

$$F_1 (x_1 + x_2) - (F_1 + F_2) x_2 = 0$$

$$F_1 (x_1 + x_2) = (F_1 + F_2) x_2$$

$$\underline{\frac{x_2}{x_1 + x_2} = \frac{F_1}{F_1 + F_2}}$$

Example #2 fulcrum at one end



Consider two forces F_1 and F_2 acting on a beam of negligible mass. One force acts vertically downwards near the centre, while the other acts vertically upwards at the end.

When the beam is balanced, the sum of the downward forces equals the sum of the upward forces.

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If the reaction force at the fulcrum is *R*,

$$R = F_2 - F_1$$

taking moments about O,

$$F_2(x_2 - x_1) - (F_2 - F_1)x_1 = 0$$

$$F_2(x_2 - x_1) = (F_2 - F_1)x_1$$

$$\frac{x_1}{x_2 - x_1} = \frac{F_2}{F_2 - F_1}$$

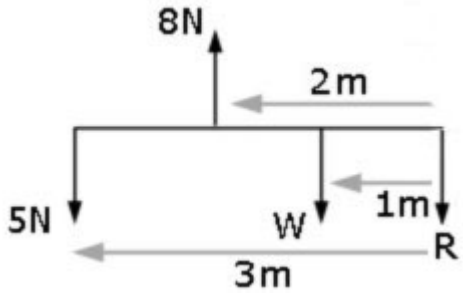
taking moments about B,

$$F_1(x_2 - x_1) - (F_2 - F_1)x_2 = 0$$

$$F_1(x_2 - x_1) = (F_2 - F_1)x_2$$

$$\frac{x_2}{x_2 - x_1} = \frac{F_1}{F_2 - F_1}$$

Example #3 a typical problem



A beam of negligible weight is horizontal in equilibrium, with forces acting upon it, as shown in the diagram.
Calculate the value of the weights R and W.

vertically $8 = 5 + W + R$

$R = 3 - W$ (i

taking moments about R

$(5 \times 3) + (W \times 1) = 8 \times 2$

$15 + W = 16$

$W = 16 - 15$

$W = 1N$

from (i

$R = 3 - 1$

$R = 2N$

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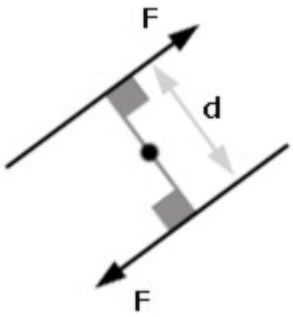
Couples



The turning effect of two equal and opposite parallel forces acting about a point equals the product of one force(F) and the perpendicular distance between the forces(d).

$$\text{couple} = F \times d$$

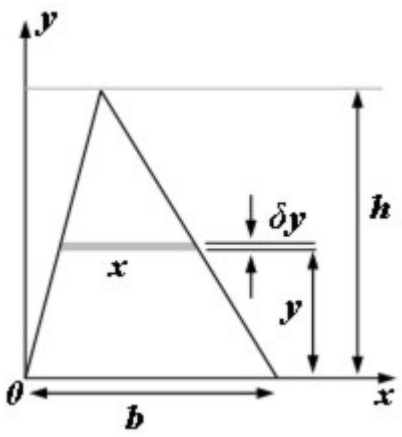
Centre of Mass

To understand the concept of centre of mass we must first consider a rigid body to be made of an infinite number of particles. Each particle has a gravitational force on it directed towards the centre of the Earth. Assuming these forces are parallel, the weight of the body equals the sum of all the tiny particle weights.

This effective weight may be considered to act through a particular point called the ***centre of mass***(sometimes termed the **centroid**)

When trying to calculate the centre of mass of a uniform solid eg a cone or hemishpere, we consider the whole mass and its moment about an axis and equate this to the sum of all the moments of constituent masses about the axis. The constituent masses often take the form of thin slices of the regular solid.

Centre of mass of a triangular lamina



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Consider a small horizontal strip of area δA(delta A).

area of small strip $\delta A = x\delta y$

by similar triangles,

$$\frac{x}{(h-y)} = \frac{b}{h}$$

$$x\delta y = \left[\frac{b(h-y)}{h}\right]\delta y$$

total area A is given by,

$$A = \frac{1}{2}bh$$

let distance of centroid from x-axis be $y_{\mathbf{c}}$

taking moments about the x-axis

area of triangle $\times$ $y_{\mathbf{c}}$ = sum of slices $\times$ slice displacements

$$\frac{1}{2}bhy_{\mathbf{c}} = \int_0^h \left[\frac{b(h-y)}{h}\right] y dy$$

$$y_{\mathbf{c}} = \frac{b}{h}\cdot\frac{2}{bh}\int_0^h (h-y)y dy$$

$$= \frac{2}{h^2}\int_0^h (hy - y^2) dy$$

$$= \frac{2}{h^2}\left[\frac{hy^2}{2} - \frac{y^3}{3}\right]_0^h$$

$$= \frac{2}{h^2}\left[\frac{h^3}{2} - \frac{h^3}{3}\right] - \frac{2}{h^2}[0]$$

$$= \frac{2}{h^2}\left[\frac{h^3}{6}\right]$$

$$\underline{y_{\mathbf{c}} = \frac{h}{3}}$$

The centre of mass is on a horizontal line a third of the height up from the base

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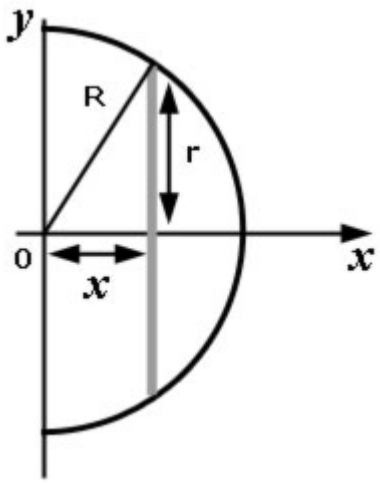
Centre of mass of a hemisphere



The method of calculation for the centre of mass of a 3D object is very similar to that of a 2D object. In this case we sum thin 2D slices.
The hemispherical axis is the x-axis and this time we consider circular slices of thickness δx.

The radius r of each slice is given by Pythagoras' Theorem:

$$R^2 = r^2 + x^2$$

hence the area of a slice is given by,

$$\Pi r^2 = \Pi(R^2 - x^2)$$

so the volume of a slice δV is given by:

$$\delta V = \Pi(R^2 - x^2)\delta x$$

Let the centroid be at position x_C from the origin O.

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$$\frac{2}{3}\pi R^3 x_c = \int_0^R \pi(R^2 - x^2)\,x\,dx$$

$$x_c = \frac{1}{\frac{2}{3}\pi R^3}\int_0^R \pi(R^2 - x^2)\,x\,dx$$

$$= \frac{1}{\frac{2}{3}R^3}\int_0^R (R^2 x - x^3)\,dx$$

$$= \frac{1}{\frac{2}{3}R^3}\left[R^2\frac{x^2}{2} - \frac{x^4}{4}\right]_0^R$$

$$= \frac{1}{\frac{2}{3}R^3}\left[R^2\frac{R^2}{2} - \frac{R^4}{4}\right] - \frac{1}{\frac{2}{3}R^3}[0]$$

$$= \frac{1}{\frac{2}{3}R^3}\left[\frac{R^4}{2} - \frac{R^4}{4}\right]$$

$$= \frac{1}{\frac{2}{3}R^3}\left[\frac{R^4}{4}\right] = \frac{3}{2R^3}\left[\frac{R^4}{4}\right]$$

$$\underline{x_c = \frac{3}{8}R}$$

Toppling

This is to do with a rigid body in contact with a rough plane. Normally the body sits on the plane in equilibrium, with the plane horizontal.

However if the plane is tilted, this state of affairs only remains until what is called the 'tipping point'. This is when a **vertical line** passing through the **centre of gravity** of the body is on the point of **falling outside** of the body's **base area**.

When the line falls outside of the base area, the body tips over.

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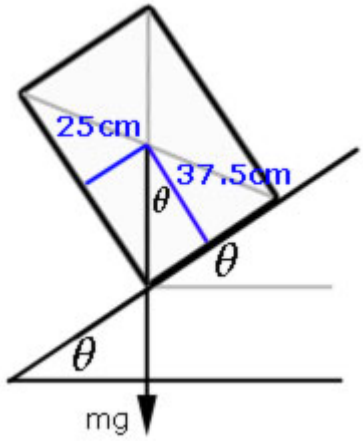
Example

A uniform cuboidal block of dimensions, height 75cm and base 30cm x 50cm rests on a rough plane, with its 50cm side up the plane.
Calculate the angle of inclination of the plane when the block is on the point of toppling(ans. to 1 d.p.)
(assume that the frictional forces on the base of the block are too high for the block to slide)



$\tan\Theta = 25/37.5$

$\Theta = 33.7^{o}$

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